Modeling Hourly Energy Use in Commercial Buildings With Fourier Series Functional Forms

Introduction

The pattern of hourly energy use in almost all commercial buildings is periodic. Some uses, such as lighting and equipment loads, vary periodically in daily and annual cycles with respect to time and do not depend on ambient temperature and other weather variables. Other uses, such as heating and cooling, are weather dependent showing variation due to weather variables such as outdoor dry bulb temperature, outdoor specific humidity and solar flux along with the time-dependent periodic variation. This behavior has been observed in the data obtained from many sites in Texas under the Texas LoanSTAR program (Claridge et al., 1991).

The nonlinear behavior of the hourly energy use in commercial buildings can be modeled using nonlinear techniques such as Fourier series functional forms, Artificial Neural Networks (ANNs) and Wavelet Analysis. While all these nonlinear techniques provide good approximations, the Fourier series functional forms are particularly suitable because the periodic pattern of variation in hourly energy use is such that the Fourier frequencies are more suitable than any other type of basis functions. This statement will be supported by comparing the results of application to actual data later in this paper.

The hourly energy data from commercial buildings are uniformly distributed when plotted in the form of a time series. It can be proved that if there are \((2n + 1)\) uniformly distributed data points within a fixed interval of \(0 < x < 2\), then there is always a unique trigonometric (Tolstov, 1962) polynomial of order \(n\) or less that takes prescribed values at these points. Such a trigonometric polynomial is also called the finite Fourier series:

\[
F(x) = a_0/2 + \sum_{i=1}^{n} [a_i \sin (kx) + b_i \cos (kx)]
\]  

The Fourier series model equations described in this paper for weather independent and weather dependent energy use are essentially variants of Eq. (1).

It is important to remove the effect of major operational changes from the data by creating subsets, which are called day-types, and then develop final model equations for each day-type. Creating the day-types and determining the final model equations for the day-types are all parts of the modeling procedure. In the following sections, the model equations and the modeling procedure are described followed by the results of these procedures applied to data collected from various buildings in Texas.

Literature Review

Weather variables, such as outdoor temperature and solar radiation, are periodic and generally show patterns similar to those exhibited by hourly energy use in commercial buildings, normally accompanied by phase shifts due to thermal inertia in the buildings. Such climatic data have been analyzed using Fourier series by several researchers (Philips, 1984; Hittle and Pedersen, 1981). The variation of the weather variables has been represented by using daily and annual Fourier frequencies.

The use of sine and cosine basis functions to describe nonlinear, periodic behavior of hourly energy use with respect to time was proposed by Pandit and Wu (1983). Seem and Braun (1991) followed by using a trigonometric form assuming a weekly periodic behavior of hourly energy use. The predictive ability of this model was relatively poor. Dhar et al. (1994) hypothesized that this was because commercial buildings undergo major operational changes from weekdays to weekends and proposed the separation of data into several groups using a simple, calendar-based day-typing technique and developing a separate model equation for each day-type. This method provided very good approximations to weather independent and weather dependent energy use in many buildings. Further developments in the use of Fourier series functional forms were (Dhar et al., 1995a and 1995b): (1) generalization of the Fourier series equation for modeling weather independent and weather dependent energy use and (2) introduction of double Fourier series functional form for a Fourier series model which represents nonlinear time and temperature dependencies. Additional details are available in Dhar (1995). These Fourier series approaches developed to model hourly energy use in commercial buildings will be described and illustrated in the following sections.

General Procedures for Fourier Series Modeling

The first step in Fourier series modeling is to perform day-typing. The entire data set is segmented into several groups according to operational characteristics of the building systems. For example, some of the lights and HVAC equipment that operate during the weekdays may be shut off during the weekends. Therefore, a simple grouping of the data set can be weekdays and weekends. The procedure for segmenting the data into multiple groups is called day-typing and the calendar as well...
3.1 Day-typing. The day-typing is started with a primary group of the entire data set based on the calendar. Weekdays, weekends and holidays, Christmas and other periods such as semester breaks in universities are initially considered. Duncan's multiple range test (Ott, 1988) is then performed. The day-groups with statistically insignificant differences in mean energy consumption are aggregated together to arrive at primary day-types. A histogram of the amplitudes of each important frequency is developed and checked for multimodal behavior. The important frequencies are those which appear consistently in a particular type of building. In general, first sine and cosine, second cosine and fourth sine frequencies are important for lights and equipment energy use in institutional buildings (Dhar et al., 1994).

Only if (1) the histogram is multimodal and (2) a physical reason can be attributed to such a distribution is it recommended that the particular day-type be divided into groups. The last step of day-typing is to repeat the Duncan test and aggregate the day-types with statistically insignificant differences in mean energy consumption.

3.2 Selection of Principal Frequencies. Fourier series functional forms are regressed for each day-type to determine the coefficients of the frequencies. However, the functional form contains a large number of frequencies and the important frequencies need to be identified to obtain model equations with fewer regressor terms. There can be several ways to do this; one popular statistical way is stepwise regression. Dhar et al. (1994) proposed the use of stepwise regression with the forward selection procedure to identify the important frequencies. The initial set of frequencies is decided by Mallow's Cp (coefficient of parameters) criteria. The value of Cp decreases with the increase in number of terms (or parameters) and becomes almost equal to the number of terms (Cp = p) at a certain point which is used as the cutoff (Ott, 1988). The number of frequencies is reduced further by using an arbitrary but reasonable partial R-square cutoff criteria (e.g., partial R-square < 0.005).

4 Fourier Analysis of Weather Independent Data

The weather independent data are modeled by a Fourier series with hour of day and day of year as the independent variables. The development of the model equation, a detailed case study and the results of several case studies are described in the following subsections.

4.1 Model Equation. The variation of hourly weather independent energy use generally shows periodicity with respect to time in the daily and annual cycles for the cases treated in the literature. The variation due to other cycles is generally not significant for most commercial buildings. A model equation that accounts for the variation in these two cycles is, therefore, presented. If the pattern shows the existence of any other set of important cycles, a similar approach can be taken to obtain the preliminary model equation with unknown coefficients. The model equation that accounts for variation in daily and annual cycles and the interaction between these two cycles Dhar et al. (1994) is as follows:

\[ E_{da} = X(d) + Y(h) + Z(d,h) + \epsilon_{da} \]

\[ X(d) = \sum_{i=0}^{\infty} \left[ \alpha_i \sin \left( \frac{2\pi}{F_i} d + \delta_i \right) \cos \left( \frac{2\pi}{F_i} d \right) \right] \]

\[ Y(h) = \sum_{i=0}^{\infty} \left[ \alpha_i \sin \left( \frac{2\pi}{F_i} h + \beta_i \right) \cos \left( \frac{2\pi}{F_i} h \right) \right] \]

\[ Z(d,h) = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \left[ \psi_{ij} \sin \left( \frac{2\pi}{F_i} d + \psi_{ij} \right) \sin \left( \frac{2\pi}{F_j} h + \psi_{ij} \right) \right] \times \left[ \eta_{ij} \sin \left( \frac{2\pi}{F_i} d + \eta_{ij} \right) \cos \left( \frac{2\pi}{F_j} h + \eta_{ij} \right) \right] \]

There are 24 hourly observations (subscript h) in a daily cycle and 365 days (subscript d) in a year which means that the values of \( \text{int}_x \) and \( \text{int}_y \) are \((365/2 - 1) = 181\) and \((24/2 - 1) = 11\) respectively. The expressions for \( P_i \) and \( P_j \) are

\[ P_i = \frac{365}{i} \quad \text{and} \quad P_j = \frac{24}{j} \]

for daily and annual cycles respectively.

4.2 A Detailed Case Study. Modeling 'Whole Building Electric' energy use \( (E_{ewb}) \) in ZEC in 1992 is presented as an example of a detailed case study. ZEC is a large institutional building containing class rooms, labs, offices and computer facilities. It should be noted that heating and cooling for this building are provided by a central campus plant; hence, \( E_{ewb} \) for ZEC does not contain these major weather-dependent uses as would be the case in most buildings.

One year of \( E_{ewb} \) data was grouped as follows by using the calendar: (i) weekdays, (ii) weekends, (iii) holidays (1st Jan, Spring break: 16th Mar., to 20th Mar., 3rd Jul., Thanksgiving: 26th to 29th Nov.), referred to as the holiday group and (iv) 26th to 29th Nov.,), referred to as the Christmas group. Duncan's multiple range test was then performed which suggested three primary day-types: (i) weekdays, (ii) weekends + holidays (will be called weekend day-type) and (iii) Christmas. The Christmas period shows separate day-type behavior in this case because the entire university closes for this nine-day period and activity level is noticeably lower than the activity levels during other holidays. The histogram of important frequencies (1st sine and cosine, 2nd cosine and 4th sine) was developed and the 1st cosine frequency suggested that the weekdays be divided into two groups (Dhar et al., 1995a). This was also physically meaningful in the sense that the days that fell in the semester group were the semester break days. The four final day-types obtained in this way were: (i) weekdays school-in-session, (ii) weekends, (iii) semester break weekdays and (iv) Christmas.

Equation (2) was then used to develop a model equation for each day-type. Mallow's \( C_p \) statistic suggested a 29-parameter model (Table 1); however, the partial \( R\)-square cutoff criteria brought the number of important terms down to seven for the weekday model equation. The \( R\)-square and Coefficient of Variance \( (C.V.) \) of the fit for the four day-types are given in Table 2. The C.V. values are between 3.8 percent and 9.5 percent indicating very good model fits. The \( R\)-square for Christmas is 0.33 which is relatively low. The data for Christmas has less scatter around the mean resulting in a poor \( R\)-square value. The C.V., however, is below 5 percent which is comparable to other day-types. The time series plots of measured and residual (measured-predicted) \( E_{ewb} \) are shown in Fig. 1.

4.3 Results From Other Case Studies. Fourier series modeling of weather independent data from eighteen sites was performed. The weekday models for different periods at different sites were studied and the results showed significant consistency with a small set of frequency terms being important. The four important terms identified for the institutional buildings weekday energy consumption are 1st sine and cosine, 2nd cosine and 4th sine frequencies. These frequencies correspond to 24 hour, 12 hour and 6 hour periods, representing day-time vs. night-time variation of energy use and the drop during lunch breaks. For more details, refer to Dhar et al. (1994).
Table 1 Summary of the forward selection procedure for whole building electricity use during school-in-session weekdays in ZEC. Data covers the calendar year 1992. Standard errors of all variables are statistically insignificant (less than $\epsilon = 0.0001$). CHi (& SHi) and CDi (& SDi) represent the $i$th frequency of the cosine (and sine) terms of the annual cycle and the annual cycle respectively.

<table>
<thead>
<tr>
<th>No. of parameters (p)</th>
<th>Variable entered</th>
<th>Partial $R^2$</th>
<th>Model $R^2$</th>
<th>Mallow's $C_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>C</td>
<td>0.8092</td>
<td>0.8092</td>
<td>43582.1</td>
</tr>
<tr>
<td>3</td>
<td>Sh</td>
<td>0.2670</td>
<td>0.2670</td>
<td>10497.9</td>
</tr>
<tr>
<td>4</td>
<td>C</td>
<td>0.0413</td>
<td>0.0413</td>
<td>5388.0</td>
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<tr>
<td>5</td>
<td>Sh</td>
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<td>0.0123</td>
<td>3863.5</td>
</tr>
<tr>
<td>6</td>
<td>S</td>
<td>0.0068</td>
<td>0.0068</td>
<td>3024.4</td>
</tr>
<tr>
<td>7</td>
<td>S</td>
<td>0.0006</td>
<td>0.0006</td>
<td>2274.2</td>
</tr>
</tbody>
</table>

The $R^2$-square and C.V. values of twenty model equations developed using data from four sites are listed in Table 2. The data for both pre-retrofit and post-retrofit periods have been modeled. The C.V. values range from 3.8 percent to 9.6 percent while the $R^2$-square values range from 0.27 to 0.96. The results in general indicate extremely good model fit. The $R^2$-square values are low for certain day-types because of less scatter of energy use around the mean for those groups.

5 Fourier Analysis of Weather Dependent Data

Weather dependent energy uses, such as hourly cooling and heating, vary in daily and annual cycles and also are affected by outdoor dry bulb temperature, outdoor specific humidity and horizontal solar flux. Two model equations, known as the Temperature-based Fourier Series (TFS) and the Generalized Fourier Series (GFS), have been developed for modeling hourly weather dependent energy use. The GFS equation uses outdoor dry bulb temperature, outdoor specific humidity and horizontal solar flux as the input weather variables assuming linear dependence, while the TFS equation uses outdoor dry bulb temperature as the weather variable, but treats nonlinear dependence. The TFS equation sacrifices little in terms of prediction accuracy when compared to the GFS equation and can be used when humidity and solar data are unavailable.

5.1 The Temperature-Dependent Model. Temperature-dependent linear (2-Parameter type) and segmented linear models (3-Parameter and 4-Parameter type) have been developed and used for modeling daily energy use in commercial buildings for retrofit savings analysis (Claridge et al., 1991). However, modeling weather dependent energy use at the hourly level should consider two important factors. (1) weather dependent energy use in commercial buildings often exhibits a nonlinear relationship with outdoor temperature. Three- and four-parameter model behavior can be observed in the data obtained from different sites (Katipamula et al., 1994, 1995) and a generic nonlinear functional form may be more appropriate. (2) Interaction between outdoor temperature variation and time (hour-of-day) due to factors such as building thermal mass and HVAC systems characteristics is significant at the hourly level. The final model equation described in this section has been developed from these perspectives.

First, we describe a Fourier series functional form to represent the nonlinear relationship between weather dependent energy use and outdoor temperature:

$$E_T = a' + b'T + \sum_{n=1}^{\infty} \left[ \alpha_n \sin \left( \frac{2\pi n (T - T_{min})}{\Delta T} \right) + \beta_n \cos \left( \frac{2\pi n (T - T_{min})}{\Delta T} \right) \right] + \epsilon_T \quad (4)$$

Table 2 Fourier model results of weather independent energy use at four educational buildings in Texas

<table>
<thead>
<tr>
<th>Building Name</th>
<th>Energy Use</th>
<th>Period</th>
<th>Data Period</th>
<th>Day-type</th>
<th>$R^2$</th>
<th>C.V. (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZEC</td>
<td>Lighting</td>
<td>Pre-retrofit</td>
<td>9/1/89 to 12/31/89 (4 months)</td>
<td>Weekdays, school-in-session</td>
<td>0.96</td>
<td>4.1</td>
</tr>
<tr>
<td>ZEC</td>
<td>Whole building electricity</td>
<td>Post-retrofit</td>
<td>1/1/92 to 12/31/92 (12 months)</td>
<td>Weekdays, school-in-session</td>
<td>0.94</td>
<td>3.8</td>
</tr>
<tr>
<td>ZEC</td>
<td></td>
<td></td>
<td></td>
<td>Weekends</td>
<td>0.94</td>
<td>3.8</td>
</tr>
<tr>
<td>ZEC</td>
<td></td>
<td></td>
<td></td>
<td>Christmas</td>
<td>0.87</td>
<td>8.4</td>
</tr>
<tr>
<td>MSB</td>
<td>Whole building electricity</td>
<td>Pre-retrofit</td>
<td>9/1/91 to 8/6/93 (35 months)</td>
<td>Weekdays, school-in-session</td>
<td>0.95</td>
<td>3.3</td>
</tr>
<tr>
<td>MSB</td>
<td></td>
<td></td>
<td></td>
<td>Weekends</td>
<td>0.95</td>
<td>3.3</td>
</tr>
<tr>
<td>MSB</td>
<td></td>
<td></td>
<td></td>
<td>Christmas</td>
<td>0.33</td>
<td>4.7</td>
</tr>
<tr>
<td>MSB</td>
<td>Whole building electricity</td>
<td>Post-retrofit</td>
<td>8/7/93 to 11/30/94 (16 months)</td>
<td>Weekdays, school-in-session</td>
<td>0.93</td>
<td>4.5</td>
</tr>
<tr>
<td>WEL</td>
<td>Whole building electricity</td>
<td>Post-retrofit</td>
<td>1/1/93 to 12/31/93 (12 months)</td>
<td>Weekdays, school-in-session</td>
<td>0.90</td>
<td>5.0</td>
</tr>
<tr>
<td>WEL</td>
<td></td>
<td></td>
<td></td>
<td>Weekends</td>
<td>0.90</td>
<td>5.0</td>
</tr>
<tr>
<td>WEL</td>
<td></td>
<td></td>
<td></td>
<td>Christmas</td>
<td>0.79</td>
<td>5.2</td>
</tr>
<tr>
<td>PAI</td>
<td>Whole building electricity</td>
<td>Post-retrofit</td>
<td>1/1/93 to 12/31/93 (12 months)</td>
<td>Weekdays, school-in-session</td>
<td>0.82</td>
<td>6.9</td>
</tr>
<tr>
<td>PAI</td>
<td></td>
<td></td>
<td></td>
<td>Weekends</td>
<td>0.82</td>
<td>6.9</td>
</tr>
<tr>
<td>PAI</td>
<td></td>
<td></td>
<td></td>
<td>Christmas</td>
<td>0.34</td>
<td>5.7</td>
</tr>
</tbody>
</table>
where \( T = (T_{\text{max}} - T_{\text{min}}) \) is the range within which the outdoor temperature varies in a particular geographic location. One can observe that the right hand side of Eq. (4) adds a temperature based Fourier series to a linear functional form of \( \alpha' + \beta' T \). In this way, Eq. (7) fits both linear and nonlinear cases efficiently.

The next step is to modify Eq. (7) to incorporate the effect of the interaction between hour of day and outdoor temperature. This is accomplished by taking the product of the time series and the temperature series which, after a few rearrangements, results in Eq. (5) described below. For further details on the derivation, see Dhar et al. (1995b).

\[
E_{T_h} = T \sum_{n=0}^{n_{\text{max}}} \left[ \phi_n \sin \left( \frac{2\pi nh}{24} \right) + \psi_n \cos \left( \frac{2\pi nh}{24} \right) \right] + \sum_{m=0}^{m_{\text{max}}} \left[ \eta_m \sin \left( \frac{2\pi m(T - T_{\text{max}})}{\Delta T} \right) \cos \left( \frac{2\pi m(T - T_{\text{max}})}{\Delta T} \right) \right] + \sum_{s=0}^{s_{\text{max}}} \left[ \gamma_s \sin \left( \frac{2\pi s h}{24} \right) + \delta_s \cos \left( \frac{2\pi s h}{24} \right) \right] + \epsilon_{T_h}
\]

The number of maximum allowable frequencies \( n_{\text{max}} \) and \( m_{\text{max}} \) depend upon the number of distinct data points within the time and temperature periods. There are 24 hours in a day, which means \( n_{\text{max}} = (24/12 - 1) = 11 \). For temperature series, \( m_{\text{max}} \) would be \((100/2 - 1) = 49\) if there were data points at 100 discrete temperature values, but results have shown that including only the first few frequencies (approximately five) in the equation are needed for modeling weather dependent energy use.

5.2 The Multivariate Model. The TFS equation is useful when only temperature data are available, the data of other weather variables (humidity and solar radiation) being unavailable because either these were not measured or the measuring instruments failed. However, if all the weather data is available, a more rational approach would be to use a model equation with all the weather variables as the regressors. The weather dependent energy use at each individual hour can be represented as a weighted linear combination of outdoor dry bulb temperature \( (T) \), outdoor specific humidity difference \( (W^+) \) and horizontal solar flux \( (I) \) (Katipamula et al., 1994, 1995):

\[
E_h = a_h + b_h T_h + c_h W^+ + d_h I_h + \epsilon_h
\]

\( W^+ \) is expressed as \( W^+ = W - W_{\text{sat}} \) where \( W_{\text{sat}} \) is the saturated specific humidity at the cold coil surface temperature of the air-conditioning system and can be assumed to be equal to 0.0092 pounds water per pound of dry air for normal air-conditioning purposes. The above equation assumes that the energy use varies linearly with outdoor temperature, humidity difference and solar flux. This assertion has been supported by a detailed study conducted by Kissock (1993), which indicated that higher order terms contribute insignificantly to the prediction and, therefore, can be neglected for practical purposes. The above equation can be generalized for all hours of the day by allowing the
coefficients to vary from hour to hour. This can be efficiently done using Fourier series. The final equation takes the following form (Dhar et al., 1995a):

$$ E_{d,h} = \sum_{k} k[X(d) + Y(h) + Z(d, h)] + \epsilon_{d,h} \tag{7} $$

where the summation includes terms for each of $k = 1, T, W$, $*$, and $l$ and $\epsilon_{d,h}$ is the error term. The functional forms of $X(d)$, $Y(h)$ and $Z(d, h)$ have been provided in section 4.1. It may be noted that while the GFS approach is intuitively more complete than the TFS approach, results discussed in subsequent sections show that the nonlinear temperature terms included in the TFS approach of Section 5.1 are often more important than the non-temperature dependent terms added in Eq. 7.

5.3 A Detailed Case Study. We will present the details of modeling cooling energy use in ZEC from Jan. through June, 1992. As stated earlier, the first step of Fourier series modeling is day-typing. Although an elaborate day-typing can be done on the residual of Eq. (6), fitted to the weather dependent data, a mere separation into weekdays and weekends has given significantly improved C.V. and $R^2$ of model fit in many cases (Dhar et al., 1995a). Weather dependent energy use depends on many factors such as building construction, HVAC plant and systems, etc., in addition to the weather variables. The pattern, therefore, is often less sensitive to minor operational changes than to weather changes. As a result, considering additional day-types, as in the case of weather independent energy use treated in Section 4.2, is not useful.

Stepwise regression was performed using TFS (Eq. (5)) and GFS (Eq. (7)) equations to identify respective sets of important parameters. The important parameters and respective partial $R$-square values for weekday and weekend cooling energy use in ZEC are given in Table 3. For the weekdays group, final GFS and TFS models each have five parameters while the final weekend models have three and seven parameters respectively.

The $R$-squares of the GFS and TFS weekday models are the same (0.89) and the C.V. values, although not identical, are quite close (12.45 percent and 12.05 percent respectively). The humidity term appears to be important in the GFS equation. Since $R$-square and C.V. values of the two weekday models are very close, it can be seen how nonlinear temperature terms in the TFS equation can represent a significant portion of the variation due to humidity. This can also be observed in the weekend models. The GFS model indicates that the most important driver of energy use during weekends is humidity (partial $R$-Square is 0.77) with model $R$-square and C.V. of 0.92 and 12.02 percent respectively, while the TFS model has $R$-square = 0.84 and C.V. = 14.18 percent. The nonlinear temperature terms of the TFS model are able to represent much of the humidity effect on energy use because of the strong multicollinearity between outdoor humidity and dry bulb temperature. This multicollinearity also means that while the models provide a good representation of the combined effects of temperature and humidity, individual coefficients (e.g., the strong humidity dependence in the GFS weekday model) may not be physically significant. The model fit of the TFS equation to ZEC cooling energy use is shown in Fig. 2.

5.4 Results from Other Case Studies. The GFS and TFS models were developed for weekdays and weekends for heating and cooling energy use and weather dependent whole building electric energy use (includes chiller electricity consumption) at five sites. Sixteen model equations were developed using the data from these five sites. $R$-squares and C.V.s of these model equations are shown in Table 4. GFS cooling energy models have slightly lower C.V.s. However, TFS though unable to model cooling energy use as accurately as GFS, nevertheless captures most of the variations due to humidity and solar effects. But the TFS provided better models of heating energy use than the GFS models, as indicated by lower $R$-squares and C.V.s in all the cases. TFS model performance was similar to that of GFS models for whole building electric energy use, the only exception being the weekends in MCC, where the TFS model is significantly better than the GFS model. This is physically reasonable since heating energy use does not depend on outdoor humidity in a significant way unless humidification is provided, and the TFS approach treats nonlinear temperature dependence, which is not treated in the GFS approach presented here.

6 Comparison With Other Nonlinear Models

The ability of nonlinear models such as TFS, ANN with a backpropagation algorithm (BPN) and WaveNet (An Artificial Neural Network with wavelet basis functions) (Dhar et al., 1995c) to predict heating and cooling energy use and chiller electric energy use were compared using data from five sites. The BPNs had (i) three input nodes feeding hour of day, outdoor temperature and day of week inputs, (ii) two hidden layers each having ten hidden nodes, (ii) one output node predicting

Table 3 Partial $R$-squares of the individual parameters of Generalized Fourier Series (GFS) and Temperature based Fourier Series (TFS) models for cooling energy use in ZEC from Jan. through June, 1992. CT1 and SH1 represent the 1st frequency of the cosine and sine terms of the diurnal cycle while CT1 and CT2 represent the temperature based first two cosine terms in Eq. 4
the weather dependent energy use, (iv) gain = 0.9, learning rate = 0.1 and bias = 0, (v) sigmoid activation function and (vi) a normalization range of 0 to 0.9 for all the inputs and output. WaveNet is a one hidden layer neural network with a variable number of hidden nodes. The number of hidden nodes of the WaveNet were optimized using the stepwise regression approach. The wavelet basis functions were generated from cubic splines (Chui, 1992).

When the model fit of the TFS equation is compared with BPN and WaveNet, the C.V. values of TFS models are found to be the lowest in 4 out of 8 cases (Table 5). Comparison of ZEC cooling and heating results for 9/1/89-12/20/89 with the results of winners in the ASHRAE Predictor Shootout I (Kreider and Haberi, 1994) competition is likewise favorable. The CVs for the top five cooling predictors ranged from 12.78 percent to 14.32 percent compared to 7.30 percent and 8.03 percent for the GFS and TFS approaches. The CVs of the top six heating predictors ranged from 15.24 percent to 31.65 percent with only one below 28.08 percent while the GFS and TFS values shown are 20.88 percent and 19.56 percent respectively. It should be noted that the Predictor Shootout I data set covered 9/1/89-12/31/89 and hence includes 11 additional days. The 11 days at the end of the year were not included in the GFS and TFS tests since that period contained two days of chilled water data that were abnormally low due to a freeze and broken chilled water lines. This probably accounts for the better values of ZEC chilled water CV for this period obtained for all of the entries in Table 5. The heating data experienced only minor impact when the chilled water lines broke, and so this comparison is probably more meaningful. It may be noted that TFS models generally have fewer parameters than other models, suggesting that TFS will often be a more accurate approach.

7 Summary and Conclusions

The Fourier series functional forms have provided excellent model fits in several cases described in this paper. However, it is important to investigate the strengths and weaknesses of these models in order to be able to provide suitable guidelines as to when these equations should be used for modeling hourly energy use in commercial buildings. Fourier series models are very suitable for modeling weather independent data unless the daily variation of energy use is very irregular, as is sometimes true in primary and secondary schools. If higher frequencies are present, then a local basis function approach, such as wavelets, will do significantly better than Fourier series. On the other hand, Fourier series will provide a better prediction with fewer terms in the model if the variation is harmonic or close to harmonic.

When modeling weather dependent energy use, the Fourier series models can be recommended in general. The variation of

<table>
<thead>
<tr>
<th>Site</th>
<th>Energy Use</th>
<th>Period</th>
<th>Day-type</th>
<th>$R^2$</th>
<th>C.V. (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZEC</td>
<td>Cooling</td>
<td>01/07/92-06/30/92</td>
<td>Weekdays</td>
<td>0.89</td>
<td>12.45</td>
</tr>
<tr>
<td></td>
<td>(GJ/h)</td>
<td></td>
<td></td>
<td>0.89</td>
<td>12.05</td>
</tr>
<tr>
<td>ZEC</td>
<td>Heating</td>
<td>02/16/91-08/12/92</td>
<td>Weekdays</td>
<td>0.85</td>
<td>17.07</td>
</tr>
<tr>
<td></td>
<td>(GJ/h)</td>
<td></td>
<td></td>
<td>0.85</td>
<td>16.75</td>
</tr>
<tr>
<td>ZEC</td>
<td>Heating</td>
<td>09/01/89-12/20/89</td>
<td>Weekdays</td>
<td>0.87</td>
<td>18.00</td>
</tr>
<tr>
<td></td>
<td>(GJ/h)</td>
<td></td>
<td></td>
<td>0.87</td>
<td>17.65</td>
</tr>
<tr>
<td>BUR</td>
<td>Heating</td>
<td>01/01/92-06/30/92</td>
<td>Weekdays</td>
<td>0.87</td>
<td>24.55</td>
</tr>
<tr>
<td></td>
<td>(GJ/h)</td>
<td></td>
<td></td>
<td>0.87</td>
<td>23.52</td>
</tr>
<tr>
<td>TCOM (Med. bldg. 1 &amp; 2)</td>
<td>Whole building electricity</td>
<td>08/31/93</td>
<td>Weekdays</td>
<td>0.85</td>
<td>11.96</td>
</tr>
<tr>
<td></td>
<td>including chiller (kWh/h)</td>
<td>08/31/93</td>
<td>Weekends</td>
<td>0.85</td>
<td>11.96</td>
</tr>
<tr>
<td>MCC</td>
<td>Heating</td>
<td>04/07/92-05/15/92</td>
<td>Weekdays</td>
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<td>10.38</td>
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<tr>
<td></td>
<td>(GJ/h)</td>
<td></td>
<td></td>
<td>0.89</td>
<td>10.02</td>
</tr>
</tbody>
</table>

Table 4 Comparison of $R^2$ and C.V. values of Generalized Fourier Series (GFS) and Temperature based Fourier Series (TFS) models of weather dependent energy use in five buildings

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weather dependent energy use is driven by a large number of parameters. However, results show that a nonlinear temperature dependent Fourier series model (TFS) gives very good predictions in most of the cases examined. TFS is a very efficient way of capturing this nonlinear variation, since only a few terms of the TFS equation are adequate to represent the variation and often give better results than other nonlinear techniques.

8 Acknowledgments

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References


