A Fourier Series Model to Predict Hourly Heating and Cooling Energy Use in Commercial Buildings With Outdoor Temperature as the Only Weather Variable

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Accurate modeling of hourly heating and cooling energy use in commercial buildings can be achieved by a Generalized Fourier Series (GFS) approach involving weather variables such as dry-bulb temperature, specific humidity and horizontal solar flux. However, there are situations when only temperature data is available. The objective of this paper is to (i) describe development of a variant of the GFS approach which allows modeling both heating and cooling hourly energy use in commercial buildings with outdoor temperature as the only weather variable and (ii) illustrate its application with monitored hourly data from several buildings in Texas. It is found that the new Temperature based Fourier Series (TFS) approach (i) provides better approximation to heating energy use than the existing GFS approach, ((ii) can indirectly account for humidity and solar effects in the cooling energy use, (iii) offers physical insight into the operating pattern of a building HVAC system and (iv) can be used for diagnostic purposes.

1 Introduction

Modeling hourly weather dependent loads in commercial buildings is useful for (i) determining retrofit savings, (ii) diagnostics purposes and (iii) acquiring physical insight into the operating patterns of the building HVAC systems. Currently available approaches for modeling hourly weather dependent loads can be broadly classified into three groups: (i) calibrated modeling approaches (Katipamula and Claridge, 1993), (ii) Artificial Neural Network (ANN) approaches (Kreider and Wang, 1991) and (iii) regression approaches. Among these, regression approaches are the simplest and demand little time for computation. Regression models are the most widely used to determine retrofit savings under the Texas LoanSTAR Monitoring and Analysis Program (MAP) (Claridge et al., 1991). There are more than one thousand channels that have been monitored under the LoanSTAR MAP and regression techniques are the obvious choice to analyze this large number of channels.

Two parameter (2-P), three parameter (3-P) (Fels, 1986) and four parameter (4-P) (Ruch and Claridge, 1992) types of regression models with temperature as the only variable are conventionally used to model heating and cooling energy consumption at the daily level (Kissock, 1993) to determine retrofit savings. However, modeling at the hourly level becomes important for purposes like data-screening, diagnostics and optimization of building energy use. The individual hourly approach or the Fourier series approach (Dhar et al., 1994a and b) can be adopted for modeling weather dependent loads at the hourly level. In the individual hourly approach, energy consumption data are binned into twenty-four hourly groups and multiple linear regression models using temperature, humidity potential (Katipamula et al., 1994) and solar radiation as the regressor variables are developed separately for each hour. The model equation is of the following form:

$$E_h = a_h + b_h + c_h W^+ + d_h I \tag{1}$$

where T is the ambient temperature, W^+ is the difference between ambient specific humidity and the saturated specific humidity at the cold coil surface temperature (Katipamula et al., 1994) and I is the global solar radiation on unit horizontal surface. The subscript h stands for the hour of the day and can be arbitrarily assumed to be 0 at midnight 1, at 1 a.m. and so on. Eq. (1) is used for predicting cooling energy use. The humidity term needs to be dropped while modeling sensible heating energy use. It may be noted that Eq. (1) does not include any nonlinear term and, therefore, may not be able to capture nonlinear relationship between weather variables and energy use.

A time series modeling approach was adopted by Dhar et al. (1994a and b) who pointed out that since hourly energy use in commercial buildings strongly periodic, a Fourier series treatment is a convenient way to model such data. The generalized model equation proposed for weather dependent energy use is of the following form:

$$E_{h,d} = \sum_k (X_k + Y_k + Z_k) \tag{2}$$

where $k = 1, T, W^+$ and I and

$$X_k = k \sum_{i=0}^{i_{\max}} \left[\gamma_{k,i} \sin \frac{2\pi}{P_i} d + \delta_{k,i} \cos \frac{2\pi}{P_i} d \right],$$

$$Y_k = k \sum_{j=0}^{j_{\max}} \left[\alpha_{k,j} \sin \frac{2\pi}{P_j} h + \beta_{k,j} \cos \frac{2\pi}{P_j} h \right] \quad \text{and}$$

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$$Z_k = k \sum_{i=0}^{i_{max}} \sum_{j=0}^{j_{max}} \left[\phi_{k,i} \sin \frac{2\pi}{P_i} d + \psi_{k,i} \cos \frac{2\pi}{P_i} d \right] \times \left[\eta_{k,j} \sin \frac{2\pi}{P_j} h + \zeta_{k,j} \cos \frac{2\pi}{P_j} h \right] \quad (3)$$

where d and h are the day of year representing the annual cycle and the hour of day representing the daily cycle respectively. X and Y in Eq. (2) are the Fourier Series representing seasonal and diurnal cycles respectively, while Z accounts for interaction effects.

A previous study found that in most cases the seasonal frequencies and the interaction terms do not appear in the final model. This is probably due to the fact that weather variables in the model inherently contain seasonal frequency information and is able to represent the relationship satisfactorily. Application to monitored data from several buildings showed that a Fourier series approach consistently provides a good fit to the measured data (Dhar et al., 1994a).

There are situations when only temperature data is available since the other weather variables were not measured or the data may be unreliable. In such cases, developing a time series model using temperature alone as the regressor variable becomes necessary. One approach is to drop the humidity and solar terms from Eq. (2) and use the truncated functional form for model development. If only diurnal frequencies are considered, the reduced model equation will be as follows:

$$E_h = \beta_0 + \sum_{j=1}^{11} \left[\alpha_j \sin \frac{2\pi}{P_j} h + \beta_j \cos \frac{2\pi}{P_j} h \right] + T_h \sum_{j=1}^{11} \left[\gamma_j \sin \frac{2\pi}{P_j} h + \delta_j \cos \frac{2\pi}{P_j} h \right] \quad (4)$$

However, a careful examination of Eq. (4) reveals that it accounts for a 2-P type of linear temperature dependence of energy use for a particular hour of day. Previous studies (Ruch and Claridge, 1992) have shown that such an assumption is invalid due to a combination of humidity effects and HVAC system behavior (Reddy et al., 1994). In this paper, we propose a temperature based Fourier series model equation which is capable of representing such effects. The new functional form is (i) able to partially capture the effects of humidity, solar radiation, HVAC system related effects, etc., in the cooling energy use model, and (ii) provides a better fit to heating energy use than the Generalized Fourier Series (GFS) approach (Dhar et al., 1994b). The model can also represent different linear or non-linear behavior of energy use with respect to temperature from hour to hour with only a few parameters appearing in the final model. The power of the model is illustrated by applying it to the monitored data from several buildings.

2 Model Concept and Development

The development of a Temperature based Fourier Series model involves considering first the variation of heating and cooling energy use with respect to outdoor temperature during the year over one individual hour of day, and then accounting for the variation from hour to hour in a day. Before going into the detailed formulation, it will be appropriate to discuss how a Fourier series functional form can be used to represent the variation of weather dependent loads with respect to temperature.

Historically, monitored building energy data indicated that 2-P, 3-P, 4-P, etc., functional forms can be used to model weather dependent loads. A function $f(x)$ will have the following expressions for 2-P, 3-P, 4-P, 5-P, and 6-P cases:

$$f(x) = a + bx \quad \text{2-P model} \quad (5a)$$

$$f(x) = \begin{cases} a_1 & \text{for } x \le x_{CP} \\ a_2 + b_2 x & \text{for } x > x_{CP} \end{cases} \quad \text{3-P model} \quad (5b)$$

$$f(x) = \begin{cases} a_1 + b_1 x & \text{for } x \le x_{CP} \\ a_2 + b_2 x & \text{for } x > x_{CP} \end{cases} \quad \text{4-P model} \quad (5c)$$

$$f(x) = \begin{cases} a_1 & \text{for } x \le x_{CP1} \\ a_2 + b_2 x & \text{for } x_{CP1} < x \le x_{CP2} \\ a_3 + b_3 x & \text{for } x > x_{CP2} \end{cases} \quad \text{5-P model} \quad (5d)$$

$$f(x) = \begin{cases} a_1 + b_1 x & \text{for } x \le x_{CP1} \\ a_2 + b_2 x & \text{for } x_{CP1} < x \le x_{CP2} \\ a_3 + b_3 x & \text{for } x > x_{CP2} \end{cases} \quad \text{6-P model} \quad (5e)$$

Nomenclature

CHi = i^{th} cosine frequency of Fourier time series
CTi = i^{th} cosine frequency of Fourier temperature series
C.V. = Coefficient of Variance based on root mean square error of a model
d = Day of year
E = Weather dependent energy use
$f(x)$ = A function of x
h = Hour of day
I = Global solar radiation on an unit horizontal surface
k = An index denoting series corresponding to internal load or weather variables
m = Fourier frequency of temperature series
n = Fourier frequency of hour of day series
P_j = Time period of a Fourier series
SHi = ith sine frequency of Fourier time series
STi = ith sine frequency of Fourier temperature series
T = Ambient dry bulb temperature
T_{max} = Maximum ambient temperature at a particular location
T_{min} = Minimum ambient temperature at a particular location
W = Specific humidity
x = An independent variable
x_{CP} = A change point where the slope of the model changes
X = Fourier time series for seasonal cycle
Y = Fourier time series for diurnal cycle
Z = Fourier time series for seasonal and diurnal cycle

Greek Symbols

$\alpha, \beta, \alpha', \beta'$ = Coefficients of Fourier series
$\gamma, \delta, \phi, \psi$ = Coefficients of Fourier series
η, ζ = Coefficients of Fourier series

Subscripts

cw = cooling energy use
d = day of hour
h = hour of day
hw = heating energy use
m = temperature frequency
n = time frequency
wbe = weather dependent whole building electric energy use

Table 1 Contribution of Fourier frequencies to the final model for cooling energy use at two sites from January 1, to June 30, 1992. Shi and Chi are Fourier sine and cosine hourly time frequencies, whereas Siti and Citi are Fourier sine and cosine temperature frequencies respectively (see Eq. (9))

		Cooling energy use (GJ/h) in ZEC		Heating energy use (GJ/h) in BUR	
Day-type	Variable	Partial R^2	Cumulative R^2	Partial R^2	Cumulative R^2
Weekdays	T	0.8149	0.8149	0.8370	0.8370
	ST1	0.0537	0.8685	0.0352	0.8722
	SH1	—	—	0.0135	0.8857
	CH1	—	—	0.0137	0.8994
	ST1*SH1	0.0190	0.8876	—	—
	T*CH1	0.0070	0.8946	—	—
Weekends	T	0.6936	0.6936	0.8168	0.8168
	CT1	0.0463	0.7400	0.0355	0.8523
	SH1	—	—	0.0282	0.8805
	CH1	—	—	0.0151	0.8956
	ST1*SH1	0.0681	0.8080	—	—
	T*SH1	0.0173	0.8253	—	—
	T*CH1	0.0107	0.8360	—	—
	CT2*SH1	0.0078	0.8438	—	—

Figure of typical temperature based 3-P, 4-P and 6-P models are shown in the appendix. However, a generalized n-parameter (n-P) model representation is analogous to a Fourier series model such as:

$$f(x) = \alpha + \beta x + \sum_{m=1}^{m_{\max}} \left[\alpha_m \sin \frac{2\pi}{P_m} x + \beta_m \cos \frac{2\pi}{P_m} x \right] \quad (6)$$

where α and β are the coefficients and subscript m stands for frequency. In order to generate a model within the temperature range of $T_{\min}$ to $T_{\max}$, Eq. (6) will take the following form:

$$E_T = \alpha' + \beta' T + \sum_{m=1}^{m_{\max}} \left[\alpha_m \sin \frac{2\pi m}{\Delta T} (T - T_{\min}) + \beta_m \cos \frac{2\pi m}{\Delta T} (T - T_{\min}) \right] \quad (7)$$

where

$$\Delta T = T_{\max} - T_{\min},\ x = \frac{T - T_{\min}}{\Delta T},\ \alpha' = \alpha - \frac{\beta T_{\min}}{\Delta T}$$

$$\text{and } \beta' = \frac{\beta}{\Delta T}.$$

Eq. (7) has been fitted to a set of 3-P, 4-P and 6-P models and the results are summarized in the appendix. It is also noted that a few frequencies of Eq. (7) are able to adequately represent 3-P, 4-P and 6-P models.

Eq. (7), therefore, is able to represent a functional form of weather dependent energy consumption for a particular hour using temperature as the only variable. However, the relationship between weather dependent energy use and ambient temperature may vary from hour to hour throughout a day depending on operating pattern of the building HVAC system. A generalized equation of energy use for all hours of the day can be obtained by multiplying the right hand side of Eq. (7) by Fourier series using hour of day as the variable. The model equation then takes the following form:

$$E_{T,h} = \left[\alpha' + \beta' T + \sum_{m=1}^{m_{\max}} \left[\alpha_m \sin \frac{2\pi m}{\Delta T} (T - T_{\min}) + \beta_m \cos \frac{2\pi m}{\Delta T} (T - T_{\min}) \right] \right] \times \sum_{n=1}^{n_{\max}} \left[\gamma_n \sin \frac{2\pi n}{24} h + \delta_n \cos \frac{2\pi n}{24} h \right]$$

Table 2 Comparison of R-square and C.V. RMSE values of Generalized Fourier Series (GFS) and Temperature based Fourier Series (TFS) models of weather dependent energy use in several buildings

Site	Type of Energy use	Period	Day-type	R-square		C.V. RMSE (%)	
				GFS	TFS	GFS	TFS
ZEC	E_{cw} (GJ/h)	01/07/92 - 06/30/92	Weekdays	0.89	0.89	12.45	12.05
			Weekends	0.89	0.84	12.02	14.18
ZEC	E_{cw} (GJ/h)	09/01/89 - 12/20/89	Weekdays	0.87	0.83	7.7	8.42
			Weekends	0.91	0.89	6.3	7.07
TDH (Lab & Main)	E_{cw} (GJ/h)	02/16/91 - 08/12/92	Weekdays	0.85	0.81	17.07	18.75
			Weekends	0.82	0.80	17.85	18.02
TDH	E_{hw} (GJ/h)	02/16/91 - 08/12/92	Weekdays	0.81	0.82	20.96	20.60
			Weekends	0.73	0.80	24.55	21.32
ZEC	E_{hw} (GJ/h)	09/01/89 - 12/20/89	Weekdays	0.90	0.91	20.8	19.37
			Weekends	0.87	0.89	21.1	20.1
BUR	E_{hw} (GJ/h)	01/01/92 - 06/30/92	Weekdays	0.87	0.90	17.03	14.94
			Weekends	0.86	0.90	16.59	14.57
TCOM (Med. bldg. 1 & 2)	E_{wbe} plus chiller (kWh/h)	06/01/93 - 08/31/93	Weekdays	085	0.83	6.17	6.70
			Weekends	0.46	0.47	8.68	8.64
MCC	E_{wbe} plus chiller (kWh/h)	04/07/92 - 05/15/92	Weekdays	0.89	0.90	13.88	13.02
			Weekends	0.80	0.92	11.96	8.69

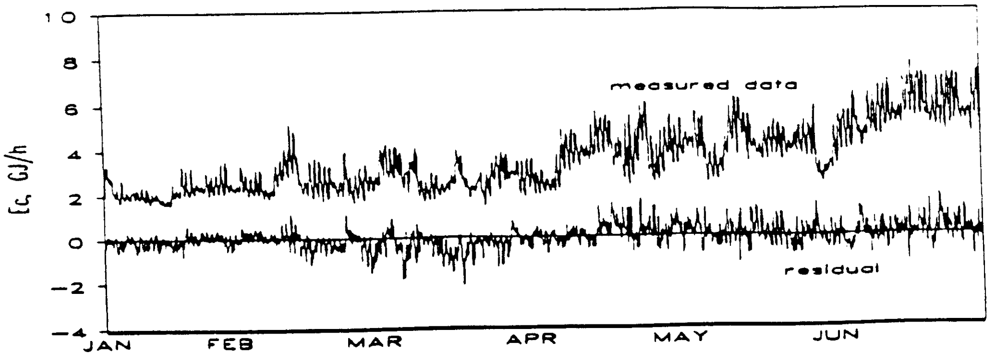

Fig. 1 Time series plots of measured and residual cooling energy use in ZEC from January through June, 1992

$$\Rightarrow$$

$$E_{T,h} = \left[\alpha' + \beta' T - \beta_0 + \sum_{m=1}^{m_{\max}} \left[\alpha_m \sin \frac{2\pi m}{\Delta T}(T - T_{\min}) + \beta_m \cos \frac{2\pi m}{\Delta T}(T - T_{\min})\right]\right] \times \sum_{n=1}^{n_{\max}} \left[\gamma_n \sin \frac{2\pi n}{24} h + \delta_n \cos \frac{2\pi n}{24} h\right]$$

$$\Rightarrow$$

$$E_{T,h} = \beta' T \sum_{n=0}^{n_{\max}} \left[\gamma_n \sin \frac{2\pi n}{24} h + \delta_n \cos \frac{2\pi n}{24} h\right] + \left[\alpha' - \beta_0 + \sum_{m=0}^{m_{\max}} \left[\alpha_m \sin \frac{2\pi m}{\Delta T}(T - T_{\min}) + \beta_m \cos \frac{2\pi m}{\Delta T}(T - T_{\min})\right]\right] \times \sum_{n=0}^{n_{\max}} \left[\gamma_n \sin \frac{2\pi n}{24} h + \delta_n \cos \frac{2\pi n}{24} h\right] \quad (8)$$

$$\Rightarrow$$

$$E_{T,h} = T \sum_{n=0}^{n_{\max}} \left[\phi_n \sin \frac{2\pi n}{24} h + \psi_n \cos \frac{2\pi n}{24} h\right] + \sum_{m=0}^{m_{\max}} \left[\eta_m \sin \frac{2\pi m}{\Delta T}(T - T_{\min}) + \zeta_m \cos \frac{2\pi m}{\Delta T}(T - T_{\min})\right] \times \sum_{n=0}^{n_{\max}} \left[\gamma_n \sin \frac{2\pi n}{24} h + \delta_n \cos \frac{2\pi n}{24} h\right] \quad (9)$$

Note that ΔT in Eq. (9) represents the range within which ambient temperature varies in a particular location. In order to avoid high extrapolation error, the data set from which the model is to be developed, should have representative uniform data density over the range of ΔT.

Eq. (9) is the Temperature based Fourier Series model that can be used to predict weather dependent loads in commercial buildings. Although Eq. (9) suggests a large number of parameters, application to monitored data from several buildings has shown that only a few terms are significant. The significant frequencies are selected from the set of terms suggested from Eq. (9) by performing stepwise regression. The model described by Eq. (9) will be called the Temperature based Fourier Series (TFS) model.

3 Application to Monitored Data

The first step in data processing prior to model identification is to day-type the data in order to remove the effect of major changes in operating schedule during weekdays, weekends, holidays, etc. Although a mere separation of data into weekdays and weekends may produce very good fits (Dhar et al., 1994a), one might adopt a rigorous day-typing technique, when necessary. Details of a day-typing procedure are described by Dhar et al. (1994b). Once day-typing is performed, separate models

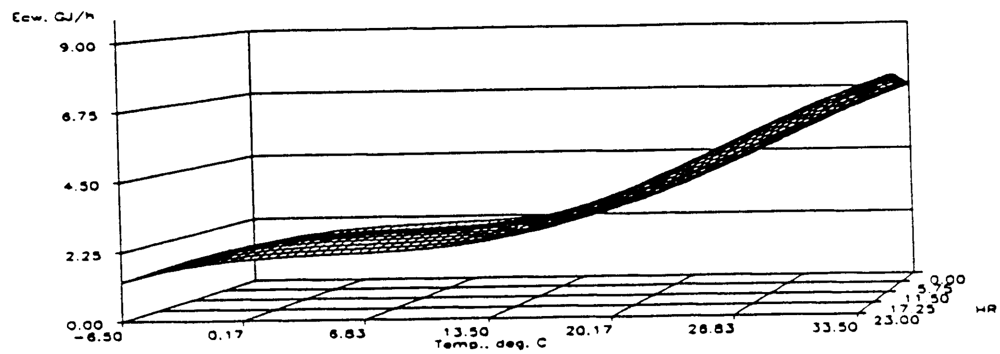

Fig. 2 A three dimensional plot of predicted cooling energy use versus hour of day and ambient temperature during working weekdays for January through June, 1992 in ZEC

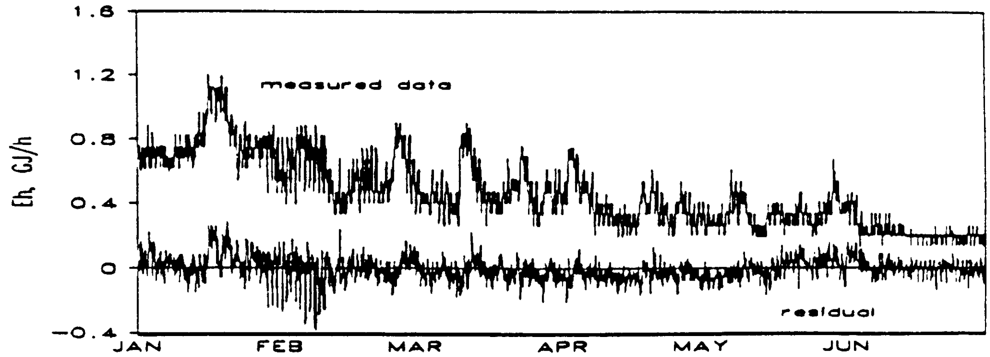

Fig. 3 Time series plots of measured heating energy use from January through June, 1992 in BUR. The residual plot after modeling with Eq. (9) is also shown. Note the relatively higher negative residual values from the end of January to mid February which identified the improper valve operation of the heating system during that period

are developed for each day-type. The usefulness of the TFS model is illustrated by examples of (1) cooling energy use during working weekdays and weekends of January 1992 to June 1992 in ZEC, a large institutional building that houses classrooms, labs, offices and computer facilities on the Texas A&M University campus, and (2) heating energy use in BUR, another institutional building (classrooms, lecture halls, office and auditorium) on the UT Austin campus. In addition, the results of applying the TFS to several other channels from different sites are also discussed.

3.1 Cooling Energy Use. Stepwise forward regression for cooling energy use in ZEC was performed using Eq. (9) in order to select the significant independent parameters from the entire set of possible parameters. Values of maximum and minimum ambient temperatures in College Station range from −6.7 degree C ($\approx$20 degree F) to 33.8 degree C ($\approx$100 degree F). A statistical criterion for selecting the significant parameters is to use Mallow's $Cp \approx p$ criterion (Ott, 1986), where p is the number of parameters. However, this criterion often retains a large number of parameters in the final model, many of which have insignificant partial R-square values. An arbitrary but reasonable cut-off of 0.005 partial R-square may be adopted in such cases to simplify the model without undue sacrifice in prediction accuracy (Dhar et al., 1994b).

The results of stepwise regression for weekdays and weekends are summarized in Table 1. The interaction terms between temperature and hour of day appear to be significant for both weekday and weekend cooling energy use with the interaction effect higher during weekends. This is probably due to the smaller variation in internal loads and occupancy during weekends which makes the variation in cooling energy use primarily temperature dependent during weekends. The first and second temperature frequencies contributed a partial R-square of 0.0697 during weekdays and 0.1502 during weekends which are about 7.7 percent and 17.9 percent of the model R-squares of 0.8946 and 0.8438 respectively. This illustrates the relevance of considering temperature frequencies and their interaction terms in the TFS model equation. The C.V. RMSE of the models during weekdays and weekends are 12.45 percent and 14.18 percent respectively (Table 2). The model fit to measured data can be seen in the time series plots of measured and residual (defined as measured use—modeled use) energy use in Fig. 1.

A three dimensional plot of predicted cooling energy use versus ambient temperature and hour of the day for weekdays is shown in Fig. 2. The plot shows how the model has captured the effects of factors like humidity and HVAC system related effects above a temperature of about 16°C.

3.2 Heating Energy Use. A similar approach was adopted to identify models for heating energy use during weekdays and weekends in BUR. The period chosen is from January 1 to June 30, 1992. The results of stepwise regression shown in Table 1 indicate that temperature frequencies have been useful for modeling heating energy use. Heating energy use in BUR, not being dependent on humidity, the TFS model gives better fits to measured data than the GFS approach (Table 2). The time series plots of measured heating energy use and residuals are shown in Fig. 3. It is noted that the model fit the data well throughout except the last week of January to second week of February. It is obvious from the time series plot of measured energy use that something unusual happened during this period. Site personnel confirmed that this was due to improper valve operation of

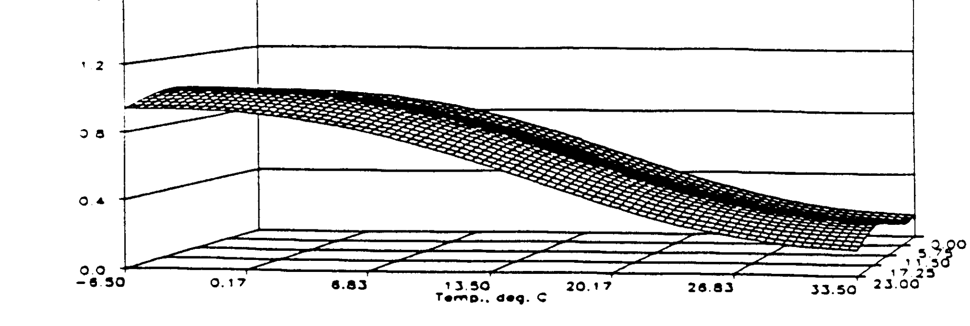

Fig. 4 A three dimensional plot of predicted heating energy use versus hour of day and ambient temperature during working weekdays for January through June, 1992 in BUR

the heating system. The time series plot of residuals generated by TFS model of heating energy use captures this unusual pattern conveniently. This illustrates how the model can be useful for diagnostic purposes.

A three dimensional plot of heating energy use in BUR during weekdays versus hour of day and ambient temperature is shown in Fig. 4. The plot shows that heating energy use drops faster with decrease in ambient temperature from 5°C and 20°C than it does at lower and higher temperatures. The daily profile does not change with temperature which means that there is no interaction between hour of day and temperature (see Table 1).

4 Comparison with Generalized Fourier Series Approach

Both Temperature based Fourier Series (TFS) models and Generalized Fourier Series (GFS) models were developed for weekdays and weekends for cooling energy use, heating energy use and weather dependent whole building electric energy use (includes chiller electricity consumption) at five sites in Texas. The *R*-square and C.V. values of these models are summarized in Table 2. It may be noted that *R*-squares are a little higher and C.V. a little lower for GFS models when applied to cooling energy use models. Thus TFS, though not able to model cooling energy use as accurately as GFS, nevertheless, captures partially the variation due humidity, solar and HVAC system related effects. However, TFS models for heating energy use are better than GFS models as indicated by higher *R*-square and lower C.V. in all the cases. TFS approach provided better results because heating energy use for the buildings under study were not dependent upon humidity and exhibited relatively high nonlinear temperature dependence. TFS models performed comparably to GFS models for whole building electric energy use in three of the four cases examined, the exception being during weekends in MCC, where TFS is significantly better than GFS. This comparison indicated that TFS is a useful modeling approach.

5 Advantages Over Existing Regression Models

The existing regression techniques such as 2-P, 3-P, 4-P, etc., models are simple and easy to apply. However, these techniques ignore (i) time dependence of energy use due to scheduling and (ii) any nonlinear relationship that might exist between energy use and ambient temperature. The TFS model proposed in this paper has the advantage of being able to represent nonlinear time- and temperature-dependence of heating and cooling energy use in a compact linear functional form. The modeling procedure gives a smooth nonlinear fit to the pattern automatically by using relevant statistical criteria.

6 Conclusions and Future Directions

The new Fourier series approach with outdoor temperature as the only weather variable has been found to model heating and cooling energy use in commercial buildings accurately. Its power lies in its ability to treat nonlinear temperature dependence by using temperature frequency terms and terms that account for interactions between hour of day and temperature. The TFS approach is found to be able to capture partially the effect to humidity and solar radiation on cooling energy use while yielding a better fit to heating energy use than the GFS approach which does not consider nonlinear temperature dependence. The model can be useful for gaining insight into the operating pattern of building HVAC systems and may be used for diagnostic purposes. However, one final point needs to mentioned. TFS approach is very effective when temperature data is available for modeling and prediction. If humidity and solar data are also available, one should prefer GFS approach for modeling cooling energy use because of the engineering relevance and higher prediction power the GFS approach offers. For modeling heating energy use, choice of either GFS or TFS approach will depend upon which functional form is a better representative of heating energy use for the particular building from both prediction and engineering perspectives. The TFS approach needs to be applied to more sites to verify its suitability to model heating and cooling energy use in commercial buildings in other parts of the world. The extension of this method to applications such as data-screening and fault diagnostics is an interesting future topic and is being currently investigated.

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APPENDIX

In this appendix we illustrate with simple examples how a temperature based Fourier series (Eq. (7)) can represent 3-P (Fig. A1) 4-P with low slope change (Fig. A2), 4-P with high slope change (Fig. A3) and 6-P models conveniently with a few frequencies. The set of equations that has been fitted with temperature based Fourier series is given in Table A1. The coefficients of these equations are so chosen as to represent the typical patterns observed in many sites in Texas (Kissock, 1993). The results of stepwise regression using the variables suggested by the Temperature based Fourier Series (TFS) model have been summarized in Table A2. It is noted that model *R*-squares and C.V. are low for all the cases, with only a few frequencies having significant partial *R*-square contribution.

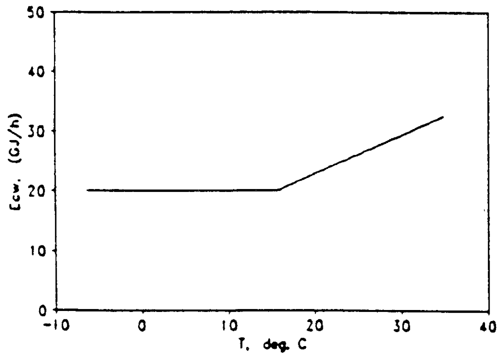

Fig. A1 Three parameter model

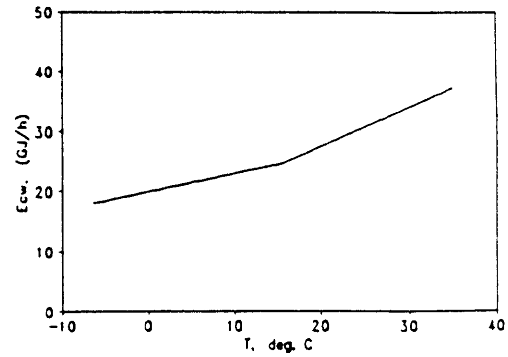

Fig. A2 Four parameter model with low slope change

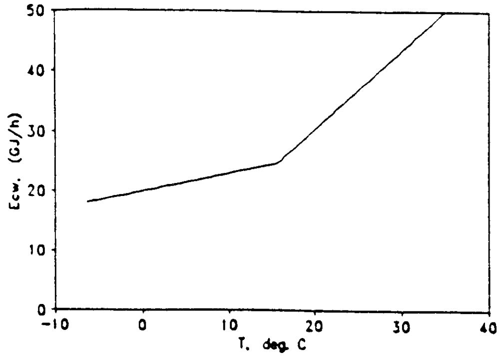

Fig. A3 Four parameter model with high slope change

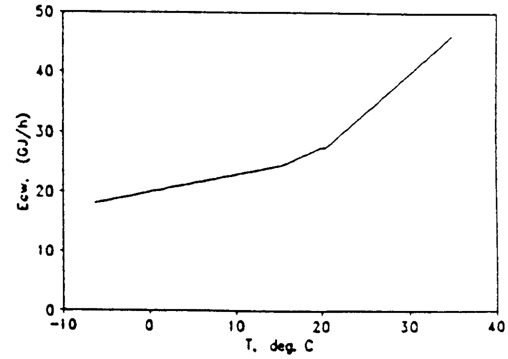

Fig. A4 Six parameter model

Table A1 Equations of examples of typical three parameter (3P), four parameter (4P) and six parameters (6P) models

Equation	Model type
$E_c = \begin{cases} 20 & \text{for } -6.67^\circ C \le T \le 15.55^\circ C \\ 9.89 + 0.65T & \text{for } 15.55^\circ C < T \le 33.77^\circ C \end{cases}$	3-P
$E_c = \begin{cases} 20 + 0.3T & \text{for } -6.67^\circ C \le T \le 15.55^\circ C \\ 14.56 + 0.65T & \text{for } 15.55^\circ C < T \le 33.77^\circ C \end{cases}$	4-P with low slope change
$E_c = \begin{cases} 20 + 0.3T & \text{for } -6.67^\circ C \le T \le 15.55^\circ C \\ 4.45 + 1.3T & \text{for } 15.55^\circ C < T \le 33.77^\circ C \end{cases}$	4-P with high slope change
$E_c = \begin{cases} 20 + 0.3T & \text{for } -6.67^\circ C \le T \le 15.55^\circ C \\ 14.56 + 0.65T & \text{for } 15.55^\circ C < T \le 21.11^\circ C \\ 0.84 + 1.3T & \text{for } 21.11^\circ C < T \le 33.77^\circ C \end{cases}$	6-P

Table A2 Results of Fourier Series model (Eq. (7)) fitted to data generated by equations shown in Table A1. Siti and Citi are the *I*th sine temperature frequencies and cosine temperature frequencies respectively

	Partial R-squares						Model	C.V. RMSE
	T	ST1	CT1	ST2	CT2	CT3	R^2	(%)
3-P	0.72	0.236	0.0289	0.0027	0.0006	0.0018	0.9912	2.23
4-P, low slope change	0.961	0.0329	0.004	0.0004	0.0002	0.0003	0.9988	1.09
4-P, high slope change	0.79	0.0108	0.1463	0.0116	0.0003	0.0204	0.9794	2.77
6-P	0.88	0.1066	0.0031	0.0052	—	0.0003	0.9953	2.71