Generalization of the Fourier Series Approach to Model Hourly Energy Use in Commercial Buildings

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Development of the accurate models for hourly energy use in commercial buildings has important ramifications for (I) retrofit savings analysis, (ii) diagnostics, (iii) on-line control and (iv) acquiring physical insights into the operating patterns of the buildings. Electric and thermal energy uses in commercial buildings, being strongly periodic, are eminently suitable for Fourier series analysis. Earlier studies assumed trigonometric polynomials with the hour of the day as the primary variable and one week as the period. This model, though suitable on the whole, was poor during certain weekday periods and during weekends. This paper presents a generalized Fourier series approach which, while ensuring a wider range of applicability, also yields superior regression fits partly because of the care taken to separate days of the year during which commercial buildings are operated differently and partly because of the rational functional form of regression model proposed. The validity of the approach is verified with year-long data of twenty-two monitored buildings.

1. Introduction

There are several incentives in modeling hourly energy use in commercial buildings: (i) to determine retrofit savings, (ii) data quality control, (iii) to understand the diurnal and seasonal nature of load shapes, (iv) to predict energy implications of change in scheduling or other retrofit measures like Energy Management and Control Systems of a building and (v) for diagnostic purposes, i.e., to identify operational and maintenance problems which increase energy use in buildings.

Energy use in commercial/institutional buildings can be subdivided into weather independent loads (like internal loads) and weather dependent loads (like comfort cooling and heating thermal energy use). Both types of energy use are strongly periodic because of systematic scheduling effects under which buildings operate and also because of the periodicity inherent in the primary forcing functions themselves. The Fourier series is a powerful and elegant mathematical tool for analyzing periodic data. Climatic data (i.e., solar radiation and outdoor temperature) are periodic and have been analyzed using Fourier series by several researchers (Philips, 1984; Hittle and Pedersen, 1981). Trigonometric models with hour of the day as primary variable have been proposed in classical literature (Pandit and Wu, 1983). Attempts at Fourier series modeling of hourly energy use in commercial buildings are relatively few, the most important being perhaps that by Seem and Braun (1991) who chose a week as the maximum period of the Fourier series. The regression fit was poor, however, partly because of the choice of the maximum period. Commercial buildings undergo major operational changes from weekdays to weekends, and a week may not be the logical period to choose.

Day-typing, i.e., identifying days over the year during which the commercial buildings are operated differently, is thus a key step in accurate modeling. An earlier study by Dhar et al. (1994) suggested that the primary day-types be determined by using the calendar method. Duncan's multiple range test is then performed in order to ascertain whether differences in mean energy use between different primary day-types are statistically significant. Although this procedure yields accurate models (Dhar et al., 1994), two short-comings were identified. Variation in diurnal energy use patterns are not explicitly accounted for since the Duncan test is performed on the total daily energy use. Also, the previous technique was not applied to weather dependent loads.

For day-typing weather independent energy use, we propose two additional steps whereby (i) data of each primary day-type is further analyzed for statistical differences in diurnal variation and (ii) Duncan test is repeated to aggregate the final day-types with statistically insignificant difference in mean energy consumption. The procedure is similar in case of weather dependent energy use, except that the residuals of a linear model with weather variables are subjected to the statistical analysis. Fourier series model coefficients are then determined separately for each final day-types.

In this paper, we present the generalized Fourier series approach of modeling hourly energy use in commercial buildings with the features discussed above. Illustrative examples are also presented.

2. Periodicity in the Data

As mentioned earlier, weather independent and weather dependent energy use have considerable periodicity diurnally and seasonally. The seasonal periodicity is illustrated by time series plots in Fig. 1 which shows whole building electricity use (E_{wbe}) over six months for a large university building in central Texas. E_{wbe}, in this case, includes internal loads and electricity to operate the air-handlers but does not include any cooling or heating energy use. Note the distinct drops during the weekends, fall-spring semester break, spring break, and spring-summer semester break. Also, E_{wbe} keeps increasing slowly from the beginning of a semester and drops gradually towards the end. Because E_{wbe} also affects cooling (E_{cw}) and heating (E_{hw}) energy use, the latter contains these variations as well as those of the weather variables (see Fig. 2). Attention of the reader at this stage is drawn to Figs. 1 and 2 only to illustrate periodicity in the measured data. Residual (difference between measured and

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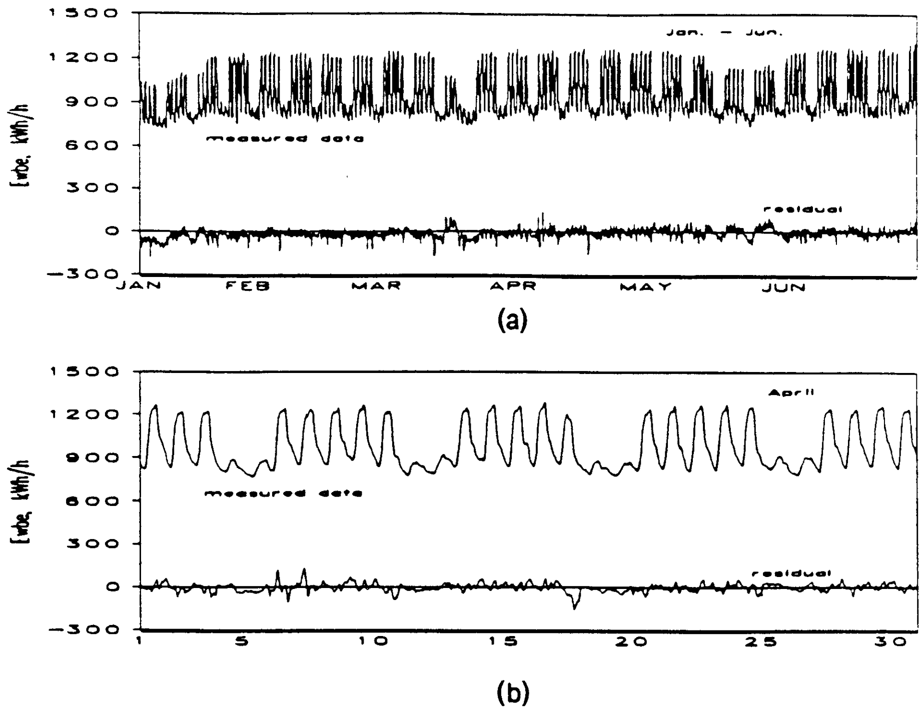

Fig. 1 **Time series plots of measured and residual whole building electricity use in ZEC in 1992**

model predicted values) time series plots are also shown in these figures to avoid duplication, and these will be discussed later in this paper.

Superimposed on the periodicity in the diurnal periodicity due to the more or less similar operating schedule of the building. The time series plot of E_{wbe} in ZEC (a large institutional building on Texas A&M University campus housing classrooms, offices, labs and computer facilities) for the month of May is shown in Fig. 1*b* to illustrate the behavior. It can be observed that during the weekdays, energy use starts increasing at around 8 a.m., dips during lunch time and drops sharply in the evening. However, the patterns are different during weekdays and weekends and, moreover, have different amplitudes. This highlights the need to perform day-typing. Time series plot of cooling energy use over the month of May is shown in Fig. 2*b* to illustrate both the weather and schedule dependencies.

3. Fourier Series Modeling

A general representation of a linear model of energy use is as follows:

$$E = \mu(h) + \epsilon \tag{1}$$

where E is the energy use, h is the independent variable and ϵ is the random error. If the variation of E with h is periodic, then $\mu(h)$ can be represented by a Fourier series (Graybill, 1976).

$$E_h = \beta_0 + \sum_{j=1}^{j_{max}} \left[\alpha_j \sin \frac{2\pi}{P_j} h + \beta_j \cos \frac{2\pi}{P_j} h \right] + \epsilon_h, \quad -\infty < h < +\infty \tag{2}$$

Nomenclature

- CDi = i^{th} cosine frequency of Fourier series for seasonal cycle
- CHi = i^{th} cosine frequency of Fourier series for diurnal cycle
- $C(p)$ = Mallow's coefficient of parameters
- C.V. = Coefficient of Variance based on root mean square error of a model
- d = Day of year, 1 on 1st January and 365 (366 for leap years) on 31st December
- E = Hourly energy use
- h = Hour of day
- I = Global solar radiation on an unit horizontal surface
- k = An index denoting series corresponding to internal load or weather variables
- m = Fourier frequency of temperature series
- p = Number of parameters of a linear model
- P_j = Time period of a Fourier series
- SDi = i^{th} sine frequency of Fourier series for seasonal cycle
- SHi = i^{th} sine frequency of Fourier series for diurnal cycle
- T = Ambient dry bulb temperature
- W = Specific humidity
- X = Fourier time series for seasonal cycle
- Y = Fourier time series for diurnal cycle
- Z = Fourier time series for seasonal and diurnal cycle

Greek Symbols

- $\alpha, \gamma, \phi, \eta$ = Coefficients of Fourier sine frequencies
- $\beta, \delta, \psi, \zeta$ = Coefficients of Fourier cosine frequencies
- ϵ = Random error
- $\mu(h)$ = A function of hour of day

Subscripts

- cw = cooling energy use
- d = day of hour
- h = hour of day
- hw = heating energy use
- le = lighting and equipment
- wbe = whole building electric energy use

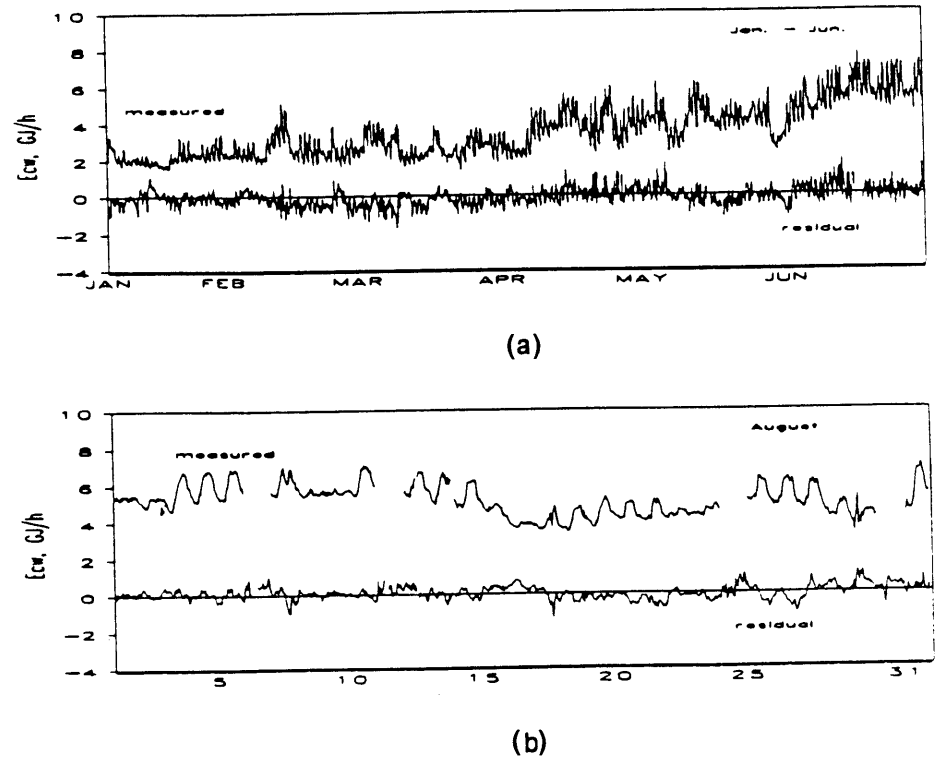

Fig. 2 Time series plots of measured and residual cooling energy use in ZEC in 1992

where E_h is the energy use at hour h, α_j and β_j are the coefficients of the j^{th} sine and cosine frequencies and P_j is the period of j^{th} frequency. An upper limit on the number of frequencies that can be chosen is $j_{max} < (\frac{24}{2} - 1)$ since otherwise, the number of parameters will be greater than the number of hours (or measured data points for the data used here) in the day. We point out the fact that a model such as Eq. (2) is restrictive in that it does not allow the mean (β_0) and amplitude (combination of α_j and β_j) to vary seasonally. Since the energy use patterns shown in Figs. 1 and 2 do exhibit such variations, a more generalized model would be of the following form:

$$E_{d,h} = X(d) + Y(h) + Z(d, h) + \epsilon_{d,h} \tag{3}$$

where

$$X(d) = \sum_{k=0}^{k_{max}} \left[\gamma_k \sin \frac{2\pi}{P_k} d + \delta_k \cos \frac{2\pi}{P_k} d \right],$$

$$Y(h) = \sum_{j=0}^{j_{max}} \left[\gamma_j \sin \frac{2\pi}{P_j} h + \delta_j \cos \frac{2\pi}{P_j} h \right]$$

and

$$Z(d, h) = \sum_{k=0}^{k_{max}} \sum_{j=0}^{j_{max}} \left[\phi_k \sin \frac{2\pi}{P_k} d + \psi_k \cos \frac{2\pi}{P_k} d \right] \times \left[\eta_j \sin \frac{2\pi}{P_j} h + \zeta_j \cos \frac{2\pi}{P_j} h \right]$$

Note that X and Y represent seasonal and diurnal periodicities respectively, while Z accounts for interaction between the two. In other words, Y alone will represent a load shape of constant mean and amplitude (a simple sinusoid in shown in Fig. 3*a*). When X is added to the expression, variation in mean energy use can also be treated (Fig. 3*b*). Addition of Z enables the model equation to represent load shapes with varying mean and varying amplitude (Fig. 3*c*). Note that seasonal variation in Eq. (3) is modeled with daily energy data. We have also investigated whether this choice is the most appropriate. For example, weekly-mean daily or monthly mean daily values of energy use could have given smoother seasonal variation patterns, yielding more accurate and robust regression models. Analysis with data from several buildings has revealed that the choice of the variable for the annual cycle does not make any significant difference. However, using daily data seems the best choice since it increases the resolution of the analysis. Appendix A illustrates this conclusion with results from model fits to data from two buildings.

Eq. (3) is the generalized regression model for weather independent energy use in commercial buildings. A suitable model for weather dependent energy use, that will incorporate the effects of both scheduling and periodicities in the weather variables, can be developed by combining the weather variables with Fourier series. While cooling energy consumption depends on outdoor temperature, solar radiation and outdoor humidity, sensible heating energy use does not depend upon outdoor humidity. Moreover, building cooling loads have two components: sensible and latent. The sensible heat gains are mainly due to the internal sensible heat load, the transmission and radiation gains through walls, roofs and windows. The latent heat gains are primarily affected by the moisture content of the fresh air intake, and also by the internal latent heat gains which are typically smaller. Instead of simply including outdoor specific humidity (W) as a variable in the model, it is more appropriate to choose the humidity difference between W and the saturated specific humidity of air at the mean cooling coil surface temperature of the HVAC system (Katipamula et al., 1994). If we assume this temperature as 12.8°C (55°F), the corresponding saturated specific humidity being 0.0092 kg per kg of dry air, then the driver of the latent load will be $W^+ = (W - 0.0092)^+$ where + signifies that W^+ should be set to zero when $(W - 0.0092)$ is negative.

Thus, a general linear equation for modeling thermal cooling energy use at each individual hour h (and for each day-type) is:

$$E_h = a_h + b_h T_h + c_h W_h^+ + d_h I_h + \epsilon_h \tag{4}$$

where T is outdoor drybulb temperature and I is the global solar radiation on a unit horizontal surface. Subscript h stands for a

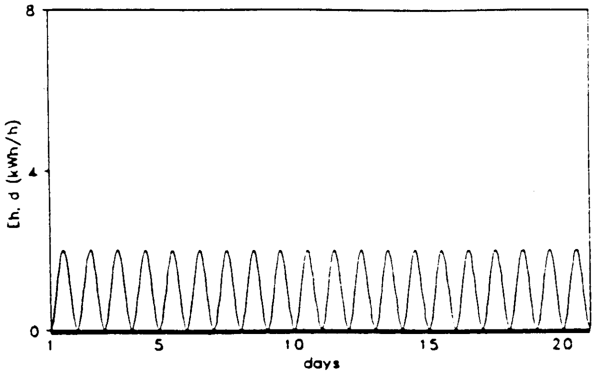

Fig. 3a Illustrative load profile for $E_{h,d} = X$ with $X = 1 - \cos((2\pi/24)h)$ showing periodic load shape with constant mean and constant amplitude

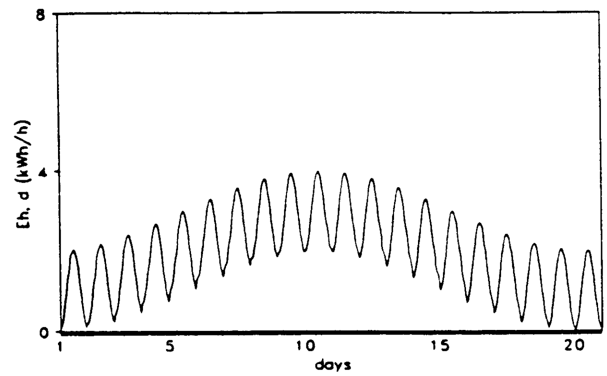

Fig. 3b Illustrative load profile for $E_{h,d} = X + Y$ with $Y = 1 - \cos((2\pi/24)d)$ showing periodic load shape with varying mean but constant amplitude

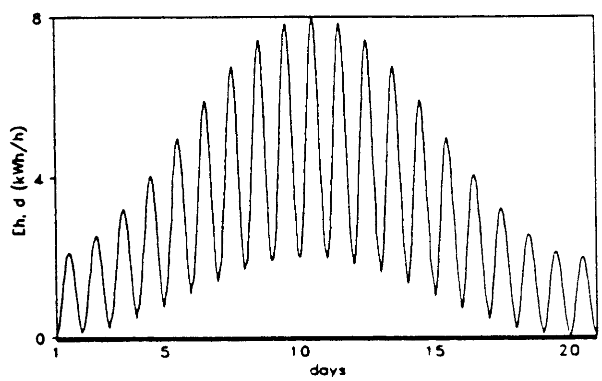

Fig. 3c Illustrative load profile for $E_{h,d} = X + Y + Z$ with $Z = XY$ showing periodic load shape with varying mean and varying amplitude

particular hour of day and can be arbitrarily assumed to be 0 at midnight, 1 at 1 a.m. and so on. When sensible heating energy use is being modeled, the coefficient c_h should be set to zero. This means that the humidity term in the right hand side of the equation should be retained if the heating energy use being modeled is used for humidification.

The coefficients of the above model vary significantly from one hour to the next over the day due to the combined effect of several factors. Internal heat gain varies according to diurnal operating schedule of the building. Overall heat transfer coefficient may vary widely over a day. Variation of b_h over a day is due to the variation in overall heat transfer coefficient and thermal lag behavior. The coefficient c_h vary over a day due to the varying infiltration or ventilation rate. Also, solar gains of the building shows different linear relationships with horizontal solar flux due to the changing position of the sun during different hours of the day and day of the year. A model such as Eq. (4), if used for all hours of day (without distinguishing between individual hours), forces the coefficients to assume mean values which are oblivious to the combined effect of the factors mentioned above. As a result, poor fits are obtained (Katipamula et al., 1994). More importantly, physical insight into the building operating schedule is lost. Again, analysis of monitored data from several Texas LoanSTAR buildings (Claridge et al., 1991) has suggested that the diurnal variation of each of these coefficients is also conveniently modeled by a Fourier series. (A case study is presented in Appendix B). Consequently, the complete general model equation for weather dependent energy takes the following functional form:

$$E_{d,h} = \sum_k k \cdot (X_k + Y_k + Z_k)$$

$$\text{where} \quad k = 1, T, W^+ \quad \text{and} \quad I \qquad (5)$$

Note that the specific humidity term needs to be omitted for sensible heating energy use models. Although Eq. (5) has a large number of terms, the data of both weather independent and weather dependent energy use support regression models with fewer terms, as will be illustrated later in this paper. The choice of which terms to retain is made based both on statistical tests and on arbitrary but realistic cut-off criteria which will be described later.

4. Day-Typing

Prior to model development, the data set needs to be divided into groups (or day-types) based on differences in operating schedule of the building systems. The method involves dividing the data set into primary day groups based on the calendar (weekdays, weekends, holiday and Christmas). Duncan's multiple range test (Ott, 1988) is then performed to aggregate to primary day-types with statistically insignificant difference in mean energy consumption. Univariate analysis of each important frequency that appear in the model of energy use is then performed separately for each day-type to divide the data further into multiple day-types. The important frequencies are those which appear consistently in a particular type of building. For example, in many university buildings, the following frequencies consistently appear for weather independent load shapes:

$$\sin\frac{2\pi}{24}h, \quad \cos\frac{2\pi}{24}h, \quad \cos\frac{4\pi}{24}h \quad \text{and} \quad \sin\frac{8\pi}{24}h,$$

The above terms correspond to the sine and cosine terms for the frequencies with 24 hours, 12 hours and 6 hours periods respectively. A histogram of the amplitude for each frequency is then developed and checked for multimodal distribution behavior. Only if (i) histogram is multimodal and (ii) a physical reason can be attributed to such a distribution, do we recommend that particular primary day-type be separated into groups. The last step of day-typing is to repeat Duncan test and aggregate the day-types with statistically insignificant difference in mean energy consumption.

The day-typing procedure described above is directly applicable to weather independent energy use. The methodology for weather dependent energy use is similar except that the effect of weather variable has to be removed first. The day-typing analysis should be performed, not on energy use but on the residuals (ϵ_h) of an equation of the following form:

$$E_h = a + bT_h + cW_h^+ + dI_h + \epsilon_h \qquad (6)$$

The complete methodology including model development has been illustrated with an example in the next section.

Table 1 Summary of forward selection procedure for whole building electric energy use during working weekdays in ZEC. Data period covers the calendar year in 1992. Standard errors of all variables are statistically insignificant (less that $F = 0.0001$). Chi (and Shi) and CDi (and SDi represent the i^{th} frequency of the cosine (and sine) terms of the diurnal cycle and of the annual cycle respectively.

No of parameters	Variable entered	Partial R^2	Model R^2	C(p)
2	CH1	0.6092	0.6092	43582.1
3	SH1	0.2670	0.8762	10497.9
4	CH2	0.0413	0.9175	5388.0
5	SH4	0.0123	0.9298	3863.5
6	SH3	0.0068	0.9366	3024.4
7	SD1	0.0061	0.9427	2274.2
8	CH3	0.0034	0.9461	1852.8
9	SD2	0.0031	0.9492	1469.0
10	SH2	0.0024	0.9516	1177.1
11	CH4	0.0022	0.9538	907.5
12	CD4	0.0016	0.9554	708.2
13	SD5	0.0016	0.9570	509.2
14	SD4	0.0011	0.9581	375.0
15	CH1*SD1	0.0005	0.9586	320.7
16	SH1*SD1	0.0004	0.9590	269.4
17	CH1*SD2	0.0004	0.9594	225.6
18	CD3	0.0003	0.9597	186.4
19	CH5	0.0003	0.9599	157.2
20	SH5	0.0002	0.9602	131.7
21	CD2	0.0002	0.9603	111.9
22	SH2*SD1	0.0002	0.9605	95.2
23	CD1	0.0001	0.9606	83.7
24	SH1*SD3	0.0001	0.9607	72.3
25	CH1*SD3	0.0001	0.9608	62.0
26	CH1*CD1	0.0001	0.9609	51.3
27	SH1*CD4	0.0001	0.9610	42.2
28	SH1*SD4	0.0001	0.9611	33.0
29	CH1*SD4	0.0000	0.9611	29.3
30	SH3*SD1	0.0000	0.9611	25.6

5. Application to Monitored Data

Twenty-two channels of prescreened hourly energy data in several Texas buildings (see Appendix C) have been modeled by using the generalized Fourier series approach. In order to illustrate the methodology and discuss the results obtained, we shall limit ourselves to monitored data of hourly whole building electricity use and cooling energy use during a complete year in the ZEC building.

5.1 Weather Independent Energy Use. ZEC is a large institutional building that contains classrooms, labs, offices and computer facilities. The seasonal and diurnal variation of whole building electric energy use are clearly seen in the time series plots shown in Figs. 1 and 2.

Primary day-types that have been developed from one year data (1992) of E_{wbe} by using the calendar method are (i) Working weekdays, (ii) Weekdays during spring break, referred to as spring break group, (iii) weekends, (iv) holidays (3^{rd} July, Thanksgiving: 26^{th} and 29^{th} November) and weekends during spring break and (v) 1^{st} January to 7^{th} January and 23^{rd} to 31^{st} December, referred to as Christmas group. An inspection of the histograms of each of the frequencies of each day-type revealed that only the first cosine frequency of working weekday group had a bimodal distribution (Fig. 4). When the data of working weekday group was divided based on the histogram, days during spring break were separated out. Finally Duncan test showed that difference between energy consumption patterns during spring break and semester break are insignificant. These two groups were, therefore, com-

Table 2 Fourier model results of weather independent energy use at two institutional buildings in Texas

Building name	Location	Energy use	Period	Data Length	Day-type	R^2	C.V.(%)
ZEC	College Station	E_{le}	Pre-retrofit	Four months	Weekdays school-in-session	0.96	4.1
					Weekends	0.64	5.1
					Christmas	0.87	8.4
		E_{wbe}	Post-retrofit	One year	Weekdays school-in-session	0.94	3.7
					Weekends	0.63	3.4
					Semester break	0.89	4.5
					Spring break	0.61	7.3
					Christmas	0.33	4.7
REC	Austin	E_{wbe}	Pre-retrofit	One year	Working weekdays	0.93	14.3
					Weekends	0.30	4.1
					Spring break	0.98	7.1
					Christmas	0.35	46.4
		E_{wbe}	Post-retrofit	Five months	Working weekdays	0.96	14.2
					Weekends	0.12	6.5
					Christmas	0.70	40.3

Table 3 Summary of Fourier series modeling for hourly cooling energy use in ZEC (entire calendar 1992) and TCOM (June '92 to August '93)

	Site : ZEC		Site: TCOM Medical bldg. 1& 2	
Day-type	Variable	Partial R-square	Variable	Partial R-square
Weekdays	INTERCEPT	---	INTERCEPT	---
	T	0.8132	T	0.6902
	W^+	0.0761	T*CH1	0.1146
	T*CH1	0.0190	SH2	0.0369
	W^+*SH1	0.0085	SH1	0.0183
	---	---	W^+	0.0364
	---	---	CH2	0.0094
	---	---	T*CH3	0.0074
Weekends	INTERCEPT	---	INTERCEPT	---
	T	0.1420	T	0.7685
	W^+	0.7780	W^+	0.0357
	---		CH1	0.0053
Spring break	INTERCEPT	---		
	T	0.5169		
	W^+	0.1230		
	W^+*SH1	0.0213		
	SH1	0.0210		
	W^+*SH2	0.0150		
	T*CH1	0.0113		
	CH2	0.0084		
	T*CH2	0.0067		

bined together. Thus, the day-typing procedure identified four day-types for weather independent energy use.

The generalized functional form suggested by Eq. (3) is used to regress hourly data of each day-type using the stepwise forward selection procedure. A widely used criteria to select the optimum number of significant frequencies is the Mallow's $C(p) \approx p$ criteria (Ott, 1988). The above procedure, when applied to model energy use, often retains a large number of terms in the final model. This is seen in Table 1 where 29 parameters (the mean or intercept term needs to be included as well) are required for $C(p) \approx p$.

A last refinement to the selection process is to drop higher frequency terms that have negligible partial R-squares. Thus if we choose an arbitrary but reasonable cut-off of 0.005 in partial R-square, then only the first 7 parameters need to be retained in the model, yielding a model R-square of 0.9427 as against 0.9611 when all 29 terms are included. However, stepwise regression results show that Z terms (Eq. (5)) were insignificant in this case, meaning that energy use during weekdays school-in-session has load shapes with fairly constant amplitude.

Table 2 presents results of applying the above procedure to monitored data of lights and equipment energy use (E_{le}) and whole building electricity use (E_{wbe}) in two institutional buildings in Texas. The regression R-squares is excellent for working weekday group which contains the largest number of days, while the R-squares are generally poorer for other groups. This is partly because of the way R-square is computed. Since R-square is a statistic describing the degree to which data scatter about the mean as explained by the model, a model fitted to data that have less scatter is likely to have a poorer R-square value. Consequently, one should also look at coefficient of variation of the RMSE to get a complete evaluation of fit. We note that coefficient of variation are low enough in most cases (Christmas period in REC building being the exception) for the models to be deemed satisfactory. The accuracy of the modeling procedure is illustrated in Fig. 1 where time series plots of measured energy use and of the residuals (i.e., the difference measured and model predicted values) are shown during the different periods of the year.

5.2 Weather Dependent Energy Use. Hourly cooling energy use (E_{cw}) was regressed by using the model Eq. (6) and the residual (ϵ_h) were subjected to frequency analysis after primary day grouping using the calendar. This involved (i) a

Table 4 Fourier series models of hourly energy use at several buildings

Building name	Type of energy use	Period	Day-type	R-square	C.V. (%)
TDh (Lab & Main bldg.)	E_{cw} (GJ/h)	02/16/91 to 08/12/92	Weekdays	0.85	17.07
			Weekends	0.82	17.85
TDH (All bldgs.)	E_{hw} (GJ/h)	02/16/91 to 08/12/92	Weekdays	0.81	20.96
			Weekends	0.73	24.55
MCC	Ewbe and chiller (kWh/hr)	04/07/92 to 05/15/92	Weekdays	0.90	12.80
			Weekends	0.93	8.58
TCOM (Med bldg. 1 & 2)	Ewbe and chiller (kWh/hr)	06/01/92 to 08/31/92	Weekdays	0.91	7.31
			Weekends	0.82	9.59
ZEC	E_{cw} (GJ/h)	01/07/92 to 12/22/92	Weekdays	0.92	10.70
			Weekends	0.91	10.50
			Spring break	0.83	8.10
ZEC	E_{cw} (GJ/h)	09/01/89 to 12/22/89	Weekdays	0.87	7.70
			Weekends	0.91	6.30
ZEC	E_{hw} (GJ/h)	09/01/89 to 12/22/89	Weekdays	0.90	20.80
			Weekends	0.87	21.10

stepwise regression of the hourly residual data in order to identify the important frequencies, (ii) generating frequency distribution for each of these important frequencies for each day-type and (iii) inspecting these distribution for multimodal behavior. We found that none of the distribution had physically consistent multimodal behavior and so the primary day-types were accepted as the final day-types.

The procedure for model development in this case is similar to that described for weather independent energy use. Again, stepwise regression was used to determine the significant terms from the set of variables suggested by Eq. (5). The model results of hourly cooling energy use in two sites (ZEC and TCOM, medical buildings 1 and 2 combined) are summarized in Table 3. The ZEC models exhibit a dramatic switch in the partial R-square contribution of T and W^+ from weekdays to weekends. This is unphysical and probably due to multicollinearity effects between both variables, a problem inherent in any multivariate regression model. The previous cut-off of partial R-square < 0.005 is also used for final selection of the set of independent variables. It is noted that standard errors of coefficients are all within an acceptable limit (probability value $\Leftarrow 0.05$) R-square values of the models for ZEC are 0.92 for weekdays, 0.91 for weekends and 0.83 for spring break, while coefficient of variation values are 10.7 percent, 10.4 percent and 8.1 percent respectively.

From Table 3 we note that none of the Z terms (see Eq. (3)) are significant. Also, in the weekday model for ZEC, the sine or cosine frequencies do not appear with significant partial R-square. However, these terms appeared for TCOM (an institutional building located in Fort Worth, Texas which houses office and classroom) due to the considerable diurnal variation of the internal load. R-square and coefficient of variation values for TCOM are 0.91 and 7.31 percent during weekdays and 0.82 and 9.69 percent during weekends.

The time series plots of cooling energy use and of the residuals in ZEC show that the models fit the measured energy use very well (Fig. 2). How the Fourier series modeling approach fares in several buildings (see Appendix C) can be gauged from Table 4. We note from R-square and C.V. values that the present Fourier series approach gives consistently good fit with R-square values being generally higher than 0.8. The four highest C.V. values are all for the heating energy use models. This is consistent with results from other models of heating energy use (Kissock et al., 1992).

6. Conclusion

The generalized Fourier series approach has been shown to be very appropriate for modeling hourly energy use data obtained from several commercial buildings in Texas. Choice of a physically meaningful day-typing technique and the rational functional form of the regression model have been instrumental in achieving consistently high prediction accuracy. A further generalization of the approach has been provided by including the interaction terms in the model equations that will be able to capture variation of both mean and amplitude, if present in the data. This ensures a wide range of applicability of Fourier series models in analyzing hourly energy use. In addition, the generalized approach offers the possibility of superior day-typing. The extent to which the Fourier series models developed in this paper accurately model energy use in other types of commercial buildings and in other parts of the world is a topic for future research. Evaluating its potential as a tool for short term forecasting and online control of energy use in buildings is also important.

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APPENDIX A

The objective of this appendix is to illustrate our conclusion that daily average energy data is the best variable of (i) daily average, (ii) weekly average and (iii) monthly average energy

Table A1 Comparison of *R*-square and C.V. Using the Fourier series models with (a) month, (b) week and (c) day as the independent variable, to account for annual periodicity of whole building electricity use

Site	Independent variable	Day-type	R-square	C.V. (%)
ZEC	month	Weekdays	0.45	4.87
		Weekends	0.43	3.65
	week	Weekdays	0.13	6.13
		Weekends	0.15	4.45
	day	Weekdays	0.57	4.32
		Weekends	0.56	3.21
BUS	month	Weekdays	0.20	13.70
		Weekends	0.33	14.72
	week	Weekdays	0.11	14.42
		Weekends	0.04	17.66
	day	Weekdays	0.37	12.18
		Weekends	0.39	14.12

use to represent seasonal variation (see Eq. (3)). One year of monitored data of E_{wbe} use in two large institutional buildings (ZEC and BUS) were taken and daily energy use was fitted with all three of these options following Eq. (2) with diurnal frequencies substituted by seasonal frequencies. From Table A1 which list the R-square and C.V. values of these three models, it is clear that the choice of daily data yields the best fits.

APPENDIX B

This appendix presents results of applying Eq. (4) to hourly monitored chilled water data during weekdays at the ZEC building September 1 to December 22, 1989. The regressions have been performed for each hour of day separately. Figures B1 and B4 illustrates how the coefficients a, b, c and d of Eq. (4) vary from hour to hour while Fig. B5 represents the R-square values and the CV-RMSE values of each hour of day. Though the patterns shown in Figs. B1 through B4 vary from building,

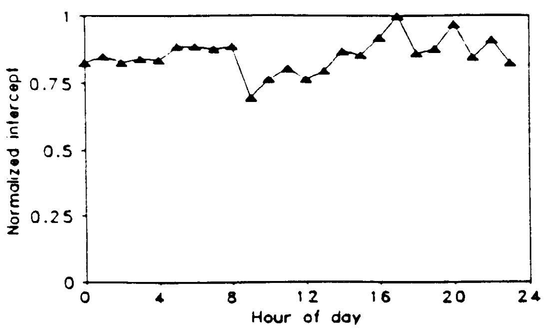

Fig. B1 A plot illustrating how the normalized intercept term (i.e., coefficient a in Eq. (4)) varies from hour to hour of the day when regressions are performed for each hour of the day separately. Data chosen is the cooling energy use in ZEC from 1st September '89 to 22nd December '89

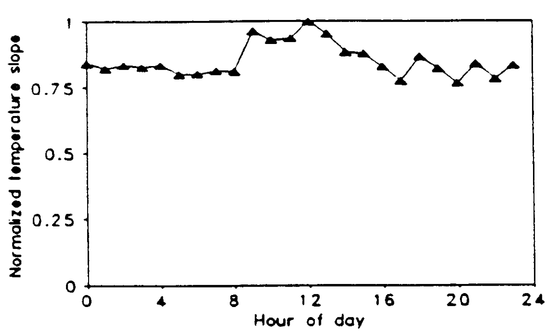

Fig. B2 A plot illustrating how the normalized temperature coefficients (i.e., coefficient b in Eq. (4)) varies from hour to hour of the day when regressions are performed for each hour of the day separately. Data chosen is the cooling energy use in ZEC from 1st September '89 to 22nd December '89

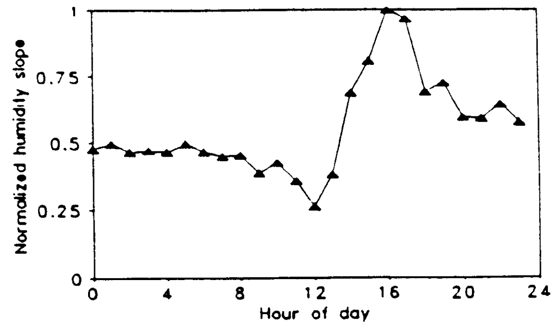

Fig. B3 A plot illustrating how the normalized humidity coefficients (i.e., coefficient c in Eq. (4)) varies from hour to hour of the day when regressions are performed for each hour of the day separately. Data chosen is the cooling energy use in ZEC from 1st September '89 to 22nd December '89

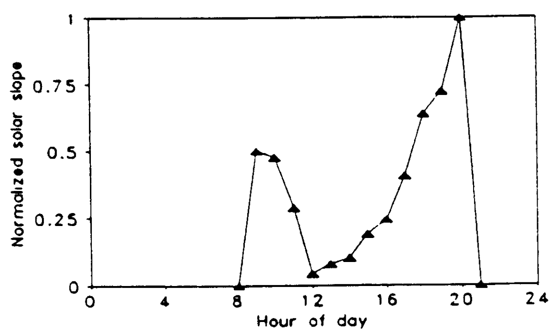

Fig. B4 A plot illustrating how the normalized solar flux coefficients (i.e., coefficient d in Eq. (4)) varies from hour to hour of the day when regressions are performed for each hour of the day separately. Data chosen is the cooling energy use in ZEC from 1st September '89 to 22nd December '89

it has been found that these diurnal variations, if modeled by low order Fourier frequency models, as used to develop Eq. (4), yield overall model fits which sacrifice but little in terms of accuracy when compared to individual hourly models. For example, the individual hourly model approach when applied to the above chilled water data yields CV-RMSE values of 7.7 percent (with 5 terms in the model) and 6.3 percent (with 4 terms in the model). The reduction in the number of model terms is substantial as the individual hour model requires four parameters for each hour of the day, i.e., 96 model parameters for the entire day.

APPENDIX C

As stated earlier, monitored data obtained from twenty-two buildings in Texas were used to verify the applicability of Fourier series modeling approach. The key descriptors of these buildings are listed in Table C1.

Table C1 Key descriptors of Texas buildings whose monitored data were analyzed during the framework of this study

Sl. No.	Building	Location	Type of Building	Area (m^2)	Type of HVAC	Type of energy use
1	EDB	Austin	Class rooms, offices	23,340	DDVAV	E_{wbe}, E_{cw}
2	UTC	Austin	Class rooms	14,190	DDVAV	E_{wbe}, E_{cw}
3	PCL	Austin	Library	44,970	SDVAV, DDVAV	E_{wbe}, E_{cw}
4	GAR	Austin	Class rooms, offices, auditorium	5,020	SDVAV, DDVAV	E_{wbe}, E_{cw}
5	GEA	Austin	Class rooms, offices, labs	5,670	SDVAV, DDVAV	E_{wbe}, E_{cw}
6	WAG	Austin	Class rooms, offices, labs	5,350	DDVAV	E_{wbe}, E_{cw}
7	WEL	Austin	Class rooms, offices, labs	40,850	DDVAV	E_{wbe}, E_{cw}
8	BUR	Austin	Class rooms, offices, lecture halls, auditorium	9,610	SDVAV, DDVAV	E_{wbe}, E_{cw}
9	NUR	Austin	Class rooms, lecture halls, lounges	8,810	SDVAV	E_{wbe}, E_{cw}
10	WIN	Austin	Class rooms, offices, theatre	10,130	SDVAV, DDVAV	E_{wbe}, E_{cw}
11	RAS	Austin	Class rooms, offices, labs	5,280	DDVAV	E_{wbe}, E_{cw}
12	PAI	Austin	Class rooms, offices, labs	11,930	DDVAV, Economizer	E_{wbe}, E_{cw}
13	WCH	Austin	Class rooms, offices, workshops, auditorium	4,550	SDVAV, DDVAV, Economizer	E_{wbe}, E_{cw}
14	ZEC	College Station	Class rooms, offices, labs, computer facilities	30,150	DDVAV	E_{wbe}, E_{cw}, E_{hw}
15	BUS	Austin	Class rooms, lecture halls	13,930	DDVAV	E_{le}
16	TDH (Lab & main bldg.)	Austin	Offices, labs	12,730	DDVAV	E_{wbe}, E_{cw}, E_{le}
17	TDH (Tower bldg.)	Austin	Offices	10,030	DDVAV	E_{wbe}, E_{cw}, E_{hw}
18	WMH	Monahan	Hospital	3,440	SDVAV	E_{wbe}, $E_{chiller}$
19	MCC	Monahan	Courthouse	8,370	SDVAV	E_{wbe}, $E_{chiller}$
20	SIM	Fort Worth	School	5,800	Rooftop units	E_{le}
21	TCOM	Fort Worth	Offices	46,097	Time clocks, Motion sensors.	E_{wbe}
22	DMS	Fort Worth	School	8,630	SDCAV	E_{le}