Engineering Uncertainty Analysis in the Evaluation of Energy and Cost Savings of Cooling System Alternatives Based on Field-Monitored Data

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ABSTRACT

The objective of this paper is to present a methodology and pertinent equations for performing an engineering uncertainty analysis on the savings due to a particular energy conservation measure (ECM) as compared to a baseline system. Though this study is geared toward building cooling systems, the methodology can be applied with minor modification to most building measurement and verification programs that involve performing measurements of the system, identifying a regression model, and using the model to extrapolate for future behavior once the ECM has been implemented. The methodology covers the case in which a nested model is used, as when short-term data are used to develop a model for building secondary thermal loads, which is then used to drive a model for chiller electricity use. Hence, the methodology treats measurement errors, model internal prediction uncertainty, and model extrapolation bias uncertainty in the framework of a nested model approach. A notable feature of this paper is that a nomograph, consisting of six separate interlinked graphs, has been generated based on the equations presented herein, whereby a user can graphically determine the final uncertainty in savings by selecting appropriate values of the various sources of uncertainty. The use of this nomograph is explained by means of an example.

OBJECTIVE AND SCOPE

Measurement and verification (M&V) programs are used to estimate energy and cost savings resulting from energy conservation measures (ECMs). These ECMs could be any of a number of measures, including modifications to the building internal loads or the HVAC system, replacement of existing equipment by more efficient equipment, or performing tune-ups or commissioning at various levels of detail. Professionals responsible for implementing M&V programs recognize that a very important allied issue is to determine the uncertainty in the savings estimate. The ability to determine this uncertainty provides both the energy professional and the building owner with a better sense of the risk involved in the stated energy savings estimate.

One of the three measurement and verification options proposed by the International Performance Measurement and Verification Protocol (IPMVP 1997) is to monitor specific end-uses for a short period (on the order of weeks) before and after implementation of an energy conservation measure. The savings are estimated as the difference between pre-ECM and post-ECM energy use normalized for factors such as weather, occupancy, etc. (see Figure 1). The objective of this paper is to present a methodology along with all relevant equations for performing an engineering uncertainty analysis of the energy

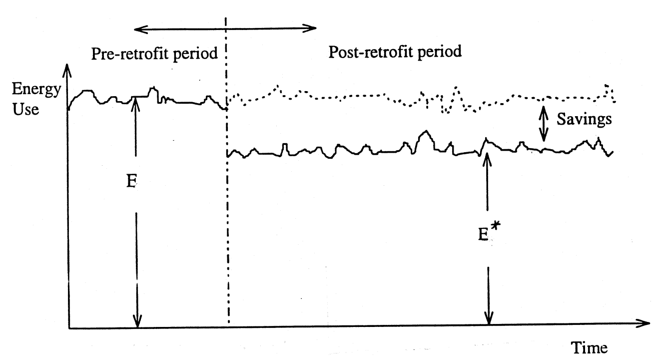

Figure 1 *Conceptual plot showing relationship between energy savings and energy use before and after the ECM.*

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savings determined from such an M&V program. This monitoring period could cover a minimum of two weeks up to a whole year. The data are to be used to generate regression models of pre-and post-ECM performance, and the models are used to estimate seasonal or annual energy and cost savings resulting from the ECMs.

The methodology was developed for evaluation of building cooling systems, and it considers elements such as building secondary loads, chillers, and auxiliary components such as pumps and cooling towers. Since this methodology is to be used by practitioners without extensive statistical and mathematical background, it is kept simple by making certain simplifying (though not simplistic) assumptions. Further, this paper is limited to engineering uncertainty issues and does not include the myriad of economic and other factors that affect estimates of future costs and savings. For example, year-to-year variability in the thermal loads and building operation and other system performance variability are not considered. Unpredictability in various economic issues, such as inflation, discount rates, energy cost, and life of the system, also are not included.

BACKGROUND

Literature Review

The need for uncertainty analysis and its methodology is well documented in the literature, and there are several textbooks that treat this subject with varying levels of detail (Coleman and Steele 1989; Dieck 1992). A proper uncertainty analysis can be very complex and cumbersome, especially if the user strives to be very meticulous. There are several references that address uncertainty analysis as applied to the evaluation of HVAC systems. *ASHRAE Guideline 2-1986* (ASHRAE 1990) provides guidelines for reporting uncertainty in results of experimental data as applied to HVAC equipment. ASHRAE RP-827 (Phelan et al. 1997a, 1997b) illustrates how to determine uncertainty bands of regression models identified from field data of fans and pumps and when measurement uncertainty is also present in the regressor variables. Appendix B of ASHRAE Standard 150P (ASHRAE 1997) provides an overview and example of uncertainty analysis for a measurement of the integrated capacity of a cool storage device or system. These references address specific issues, and none of them addresses the full scope of an uncertainty analysis for the case considered here.

Kammerud et al. (1999) formulated a more global uncertainty methodology for the economic evaluation of options considered in chilled water plant design. The methodology, which is based on life-cycle present worth analysis in terms of a benefit-cost ratio, addresses present as well as year-to-year uncertainty in economic and cost factors and in building and system performance factors and the relative importance of economic and engineering uncertainties on the overall uncertainty. However, its treatment of the sources of engineering uncertainty is simplified, and it is this area in particular upon which the current paper is focused.

Sources of Uncertainty

Any measurement has some error associated with it. Such error is the difference between the measured value and the true value. Uncertainty is the interval around the measured value within which the true value is expected to fall with some stated confidence. A statement of measured value without an accompanying uncertainty statement has limited value (Dieck 1992). The uncertainty interval gives the party who will be using the measurement result a means of assessing its value. It is especially important to perform an uncertainty analysis when making measurements that will have financial implications that may end up being examined in court.

There are three separate sources of uncertainty when using observed data to develop a predictive performance model. Consider a model such as: $y = a_0 + a_1 \cdot x_1 + a_2 \cdot x_2$ where the *x's* are the independent variables and *a*'s are model coefficients. (It is more accurate to use the term "regressor variables" instead of "independent variables" because the variables may not be independent; for example, outdoor dry-bulb temperature and outdoor humidity, which are used to model building energy use, are usually correlated to a certain extent.) An uncertainty in the variable *y* can arise from three sources:

1. measurement errors,
2. prediction uncertainty when a regression model is fit to data assumed to have no measurement error in the regressor variables, and
3. regression model prediction uncertainty when the regressor variables have inherent uncertainty/error either due to the measuring instrument or because the regressor variables themselves are determined from another regression model (a procedure called a "nested" model approach).

Each of these sources of uncertainty is described in more detail below.

Measurement Errors. If the coefficients a_0, a_1, and a_2 of the above stated model are known with zero uncertainty, the uncertainty in the derived variable *y* is only due to the measurement uncertainties present in the *x*'s. This is true if the *a* coefficients are constants or are tabulated values such as steam properties, for example. The uncertainty in *y* for such models or equations is given by the "propagation of errors" formulae with which most engineers are familiar (Kline and McClintock 1953). An example of this type of uncertainty is when the charging rate of a thermal energy storage system is deduced from measurements of mass flow rate and inlet and outlet temperature differences (ASHRAE 1997).

The sources of measurement errors can be further divided into (a) calibration errors, (b) data acquisition errors, and (c) data reduction errors (ASME 1990). These lead to essentially two types of error: a systematic or bias error and a random or precision or "white noise" error. It is usually cumbersome to perform an uncertainty analysis with data having known biases. It is far simpler to remove known biases from the data prior to data analysis and only treat random errors. However, Coleman and Steele (1989) argue that one should also explicitly include bias errors of the instrument which arises from the precision error of the primary or reference instrument against which the field instrumentation is calibrated. The authors also present pertinent formulae to treat both bias and random errors in a rigorous fashion.

Prediction Uncertainty with No Error in Regressor Variables. This error source occurs when the x's are assumed to have no error in themselves while the coefficients a_0, a_1, and a_2 have some inherent uncertainty as a result of identifying them from regression to measured data. There is prediction uncertainty in the y variable under such a case since any regression model with a goodness of fit $R^2 < 1$ is incapable of explaining the entire variation in the regressor variable. This source of error is called model internal prediction uncertainty. An example of this source of uncertainty is when a simple regression model is used to predict building cooling loads from outdoor temperature (T). If the measurement error in T is so small as to be negligible, then the uncertainty in predicting building loads falls in this category. Practitioners usually assume the model residuals to be well-behaved (i.e., have constant variance and no serial correlation) and use the resulting statistical formulae. This case is well treated in most textbooks (for example, Draper and Smith 1981) and even in the HVAC literature (for example, Phelan et al. 1997a, 1997b). In the case of improper residual behavior, the treatment gets considerably more complex, and the interested reader can refer to Reddy et al. (1998) for a more elaborate discussion as applied to building secondary loads.

The determination of prediction uncertainty from using regression models is subject to different types of problems. The various sources of uncertainty can be classified into three categories (Reddy et al. 1998):

a. *Model mis-specification uncertainty*, which arises when the functional form of the regression model does not adequately approximate the true driving function of the response variable. Typical causes are (i) inclusion of irrelevant regressor variables or non-inclusion of important regressor variables (for example, neglecting humidity effects); (ii) assumption of a linear model, when the physical equations suggest nonlinear interaction among the regressor variables; and (iii) incorrect order of the model, i.e., either a lower order or a higher order model than the physical equations suggest. Engineering insight into the physical behavior of the system helps minimize this type of error.

b. *Model prediction uncertainty*, which arises due to the fact that a model is never perfect. Invariably a certain amount of the observed variance in the response variable is unexplained by the model. This variance introduces an uncertainty in prediction.

c. *Model extrapolation uncertainty*, which arises when a model is used for prediction outside the region covered by the original data from which the model has been identified. Models identified from short data sets, which do not satisfactorily represent the annual behavior of the system, will be subject to this source of uncertainty.

The functional forms of the regression models are often well known from engineering thermodynamic and heat transfer considerations, and there are numerous statistical packages that can be conveniently used to perform the regression fits. However, the error or uncertainty associated with these regression models is often not characterized properly, either because of misinterpretation by the user of the regression results or because the statistical packages have only limited error diagnostic capabilities, i.e., inherent assumptions of how the data behaves that may be contrary to physical and actual behavior. In fact, most of the statistical packages have error diagnostics that apply only to a limited number of practical cases though often users are not sufficiently knowledgeable to discern these limitations.

Prediction Uncertainty with Error in the Regressor Variables. This type of uncertainty arises when both the regressor variables and the model coefficients have uncertainties, the former due to measurement errors or if a nested model is used and the latter because a regression model is identified from monitored data. The standard practice in classical regression analysis is to assume no measurement error in the regressor variables. Such an assumption is perhaps inadmissible for many HVAC&R applications since we may then be placing too much confidence in our predictions and underestimating the uncertainty. An example of this source of uncertainty is when a polynomial model is used to predict pump electricity consumption from measured values of fluid flow rate that inherently have non-negligible measurement errors (Phelan et al. 1997a, 1997b).

Short-Term to Long-Term Predictions of Building Thermal Loads

There are no absolute rules for determining the minimum length of the monitoring period to achieve acceptable accuracy from regression model predictions of the long-term building secondary or HVAC system loads. A full year of energy consumption data is likely to encompass the entire range of variation of both climatic conditions and the different operating modes of the building and of the HVAC system. However, in many cases a full year of data is not available and one is constrained to develop models using less than a full year of data. The accuracy with which temperature-dependent regression models of building thermal energy use identified from short data sets (i.e., less than one year) are able to predict annual energy use for large buildings in Texas has been investigated by Kissock et al. (1993) and by Katipamula et al. (1995). Further investigation has also been performed with synthetic energy use data generated from engineering models

(Reddy et al. 1998). All of these studies reached the same general conclusions:

1. Only if the monitoring is performed during the swing seasons that experience large variations in outdoor climatic conditions can one expect to have good long-term load predictions.
2. There seems to be no way of adjusting regression models to accurately predict annual energy use once improperly identified from short data sets.

These findings and the strategy suggested may, however, be unacceptable in several measurement and verification projects since one does not have the luxury of waiting until the climatic conditions are favorable to perform the in-situ tests. A cost-effective alternative option has also been suggested (Abushakra et al. 1999). The basis of this alternative is that practical considerations dictate that in-situ tests (even if they entail nonintrusive monitoring left at site with automated data collection and retrieval) cannot last several months but should be limited to two to four weeks at the most. A monitoring scheme (termed the short-term monitoring/long-term prediction—SMLP) was proposed, which involves about two to four weeks of hourly monitored data (which would provide the necessary insight into the magnitude and diurnal patterns of building use) compounded with utility bill information (which would capture the necessary seasonal and annual variations in the climatic variables that influence building energy use). A model developed from data consisting of hourly monitored data and utility bill data is likely to have better predictive ability into the future wherein climatic and operating conditions are bound to be different from those of the baseline data period. This scheme has been evaluated with monitored data from two commercial buildings and seems to provide results promising enough for continued study. For example, when this scheme is applied to the same data set used by several competitors of the ASHRAE Predictor Shootout II (Haberl and Thamilseran 1996), the results were only marginally poorer than the more elaborate prediction schemes.

Because the overall objective of a measurement and verification project for an energy conservation measure is to determine energy and demand savings, one needs to consider differential time-of-day and time-of -year rates for electricity in the analysis. In order to do so, one needs to evaluate the SMLP method by its ability to predict building loads on a time-of-day and on a seasonal basis. A preliminary analysis of this capability has also been carried out with the ASHRAE Predictor Shootout II data (Haberl and Thamilseran 1996). The year was divided into summer (from June 15 to September 15) and nonsummer months. Further, four different daily periods were considered:

- All hours of the day
- Occupied on-peak hours (noon until 6:00 p.m.)
- Occupied off-peak hours (6:00 a.m. until noon)
- Non-occupied off-peak hours (6:00 p.m. until 6:00 a.m.)

How well the SMLP model does with such disaggregation is summarized in Table 1. One notes that though the model CV-RMSE (coefficient of variation of the root mean square error) values are close to 10%, the relative MBE (mean bias errors) is generally small, less than 10% for summer months and close to zero for the nonsummer months. These numbers will be used for the model CV-RMSE and model extrapolation errors while performing the preliminary uncertainty analysis described in this paper. A graphical representation of the predictive ability of the SMLP method is given in Figure 2 in

TABLE 1
Summary of the Statistical Criteria Allowing a Comparison of the Predictive Accuracy of the SMLP Method for Building Load Prediction*

		Summer (6/15-9/15)				Nonsummer			
	Periods	No. of hours	Average Q_{bldg} (kBtu/h)	Model CV-RMSE (%)	Model extrapolation bias (%)	No. of hours	Average Q_{bldg} (kBtu/h)	Model CV-RMSE (%)	Model extrapolation bias (%)
1	Whole data period	1032	6.51	8.82	2.6	-	-	-	-
2	All hours	600	6.54	9.84	5.2	432	6.47	7.79	-0.93
3	Occupied on-peak	150	6.71	11.42	8.6	108	6.76	7.43	0.15
4	Occupied off-peak	150	6.45	6.95	12.4	108	6.23	10.67	-0.96
5	Nonoccupied off-peak	300	6.51	9.42	5.4	216	5.95	8.20	-0.50

*Assuming a 14-day monitoring period with 12 utility bills. ASHRAE Predictor Shootout II data (Haberl and Thamilseran 1996) for an engineering center have been used for various subsets of the entire data set (from RP-1004, 1998).

terms of load duration curves. The monitored data have been sorted in descending order and this is plotted both with the concurrent (i.e., unsorted) values of the model predictions as well as with the sorted values of the model predictions. Such an illustration provides a better illustration for the purposes to which the model predictions will be used during the uncertainty analysis.

PROPOSED METHODOLOGY

Assumptions

a. The discussion is directly pertinent to cost savings due to energy savings although, in principle, it could be extended to demand reduction savings.
b. Uncertainties in the economic variables as well as in the electricity unit costs are not included.
c. Analysis is limited to one year only, in that year-to-year unpredictability or uncertainty in building operation and installed base loads is not considered.
d. The bias and precision errors are treated together given that this is a preliminary uncertainty analysis (Coleman and Steele 1989).

Basic Equations

Following Reddy and Claridge (1999) and Kammerud et al. (1999),

(Annual \$savings in electricity use)
= (Annual energy \$cost incurred by the baseline system)
− (Annual energy \$cost incurred by the modified system)

$$\sum^{year} \$_{save} = \sum_{p=1}^{s} c_p \sum_{h=1}^{n_p} (E_{p,h} - E^*_{p,h}) \quad (1)$$

where

p = a particular period during the year where the electricity charges differ due to time-of-day and time-of-year differential rates imposed by the utility;
s = number of such periods during the year;
c_p = unit cost of electricity during period p;
h = hours during the year when a particular rate structure is in effect;
n_p = number of hours during the year contained in period p;
E = hourly baseline energy use, i.e., prior to installing the ECM;
E^* = hourly post-ECM energy use.

In the discussion that follows, we shall only consider one period, i.e., $p = 1$, so as to keep the presentation simple, although the methodology is easily extended to any number of periods. On an hourly basis, representative of the seasonal or annual average hourly value, Equation 1 simplifies to

$$\$_{save} = c(E - E^*) . \quad (2)$$

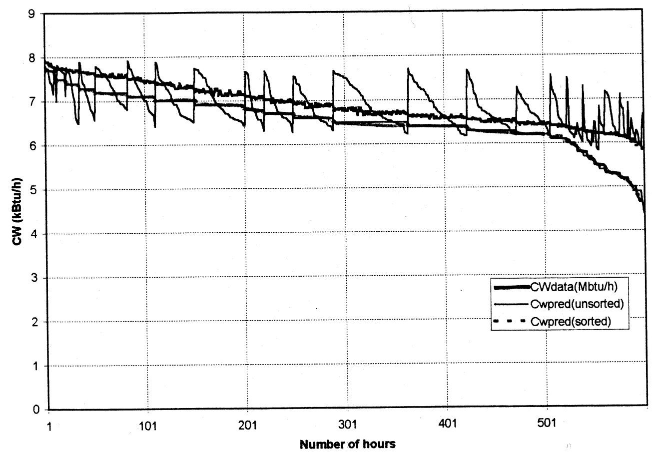

Figure 2 *Load duration curves of measured chilled-water energy use and that predicted by using the SMLP method for all hours during summer using the ASHRAE Predictor Shootout II data.*

Note that in order to keep the equations easier to follow, we have omitted the use of overbars to represent mean seasonal or annual values.

Assuming c to have no uncertainty, the uncertainty in the savings is simply

$$\Delta\$_{save} = c(\Delta E^2 + \Delta E^{*2})^{1/2} \quad (3)$$

where Δ denotes the uncertainty in either \$or E. Note that we are assuming pre- and post-ECM energy use values E and E^* to be uncorrelated. This is a simplified treatment (in keeping with the generalized treatment adopted in this paper) since these are dependent on many of the same regressor variables including those affected by the retrofit.

From the above two equations, the ratio of the uncertainty in energy savings to the magnitude of the energy savings is

$$\frac{\Delta\$_{save}}{\$_{save}} = \frac{(\Delta E^2 + \Delta E^{*2})^{1/2}}{(E - E^*)} , \quad (4a)$$

$$= \frac{\Delta E}{E} \frac{\left(1 + \frac{\Delta E^{*2}}{\Delta E^2}\right)^{1/2}}{\left(1 - \frac{E^*}{E}\right)} . \quad (4b)$$

Making a further realistic approximation that $(\Delta E^*/\Delta E) = 1$, Equation 4b simplifies into

$$\frac{\Delta\$_{save}}{\$_{save}} = \frac{\sqrt{2}}{F_{save}} \frac{\Delta E}{E} \quad (5)$$

where F_{save} is the ratio of savings divided by the baseline energy use E, i.e.,

$$F_{save} = 1 - \frac{E^*}{E} . \quad (6)$$

This is depicted schematically in Figure1. This equation provides a key insight. The energy savings fraction in most ECM projects is likely to be small (F in the range of 0.05 - 0.3). Since this term appears in the denominator, the fractional uncertainty in the dollar savings is likely to be affected more by the value of F than by the fractional uncertainty in E.

Sources of Uncertainty Described Qualitatively

Let us now present a brief description of the various sources of uncertainty that impact the evaluation of the term ($\Delta E/E$) where E is the energy use before the ECM has been implemented, which we are able to field monitor at, say, an hourly time scale. Total pre-ECM electric use E of the cooling system (excluding building related use) is

$$E = E_{aux} + E_{chiller} \tag{7}$$

where E_{aux} is the electric use by the auxiliary equipment, which includes pumps, fans, and controls, and $E_{chiller}$ is the electric use of the chiller.

Since the savings determination is based on separately identifying regression models for pre- and post-ECM, the discussion that follows is applicable to either time period. Following Phelan et al. (1997a, 1997b) and RP-1004 (ASHRAE 1998), monitored data of E_{aux} and $E_{chiller}$ can be used to identify regression models of these quantities as functions of building secondary thermal loads (Q_{bldg}) and other variables of lesser influence. In our treatment of uncertainty that follows, we shall consider the uncertainty in Q_{bldg} to be the only source of uncertainty impacting E_{aux} and $E_{chiller}$. Further, we assume that Q_{bldg} itself is predicted from a regression model with outdoor temperature T as the most influential regressor variable.

Uncertainty in E is due to the following sources:

1. Measurement error in Q_{bldg}: The precision error is already included in the CV-RMSE of the building load regression model and should not be included again. As is usual practice, we shall assume the bias error to be specified as a fraction of the full-scale reading of the BTU meter used to measure Q_{bldg}.
2. Precision uncertainty in prediction of Q_{bldg}: This uncertainty arises from the use of a regression model to predict Q_{bldg} from climatic data and building data (such as internal loads and building occupancy).
3. Bias uncertainty in the prediction of Q_{bldg}: Model extrapolation error, i.e.,the short-term to long-term bias has been discussed earlier. We shall term this as external prediction bias error. The bias error can be determined separately (or specified) for each of the periods p of the year (as shown in Table 1) or a single annual value can be used.
4. Measurement error in E_{aux} and $E_{chiller}$. The precision or random error is explicitly included in the CV-RMSE of the regression model between E and Q_{bldg}. As in the case of (1) above, we shall denote the bias error as a fraction of the full-scale instrument reading.
5. Precision uncertainty in prediction of E_{aux} or $E_{chiller}$. This arises from the use of a regression model to determine this quantity knowing Q_{bldg}. This uncertainty should be taken as due to model internal prediction error. Note that since a nested model is used (i.e., a model between E and Q_{bldg} where Q_{bldg} is itself predicted from another regression model), there is bound to be an uncertainty in internal prediction, even if we have year-long building load monitored data to develop the regression model for Q_{bldg}.
6. External prediction bias error in the prediction of E_{aux} or $E_{chiller}$. The model between these and Q_{bldg} has been identified from short-term data, and so an "external" prediction bias is likely to be present.

Uncertainty expression for Q_{bldg}

Sources of error (1), (2), and (3) described above are now considered. Error (1) is straightforward in that a bias fraction $(\Delta Q_{bldg} / Q_{bldg})_{meas,bias}$ needs to be specified. As shown in Table 2, a typical range would be from 0 to 8%. Error (2) is the prediction error of the model, assuming the model to have been identified from year-long data. We shall assume, for simplicity, that Q_{bldg} is a function of outdoor temperature T only (which is a good assumption based on several past studies (see, for example, RP-1004 [ASHRAE 1998]). Further, the small measurement uncertainty in T can be neglected. Then the uncertainty, represented by the prediction intervals (PI) of an individual prediction for a given value T_0, is given by a well-known formula available in standard textbooks (Draper and Smith 1981).

$$Q_{bldg} \pm \Delta Q_{bldg} = Q_{bldg} \pm t.RMSE\left[1 + \frac{1}{n} + \frac{(T_o - \bar{T}_i)^2}{\sum_{i=1}^{n}(T_i - \bar{T}_i)^2}\right]^{1/2} \tag{8}$$

where t is the student t statistic, which, for more than 30 observations, is close to 2 for 95% confidence level and is 1 for one standard error (68% confidence level). In the equations that follow, we shall set $t = 1$ in order not to carry t throughout all subsequent expressions.

RMSE is the root mean square error of the regression model given by

$$RMSE = \left[\sum_{i=1}^{n}(Q_{bldg,\,i} - \hat{Q}_{bldg,\,i})^2/(n-p)\right]^{1/2}. \tag{9}$$

where n is the number of data points used to identify the model, and p is the number of parameters in the model (if a

TABLE 2
Estimates of the Typical Range of Variation of Various Sources of Uncertainty and Values Used to Generate the Graphs in Figure 4

	Range of Variation	Typical Value	Description
$(\Delta Q_{bldg}/Q_{bldg})_{meas,bias}$	2-8%	4%	Mean measurement bias error of Q_{bldg}
CV_{bldg}	5-20%	10%	CV-RMSE of the regression model between Q_{bldg} and T
$(\Delta Q_{bldg}/Q_{bldg})_{model,bias}$	0-12%	6%	Model bias due to external prediction of Q_{bldg}
$(\Delta E_{elec}/E_{elec})_{meas,bias}$	0-2%	1% (*)	Mean measurement bias error of E_{elec}
$(1.26 \times CV_{chiller})/\sqrt{m}$	2-10%	6%	Effective CV-RMSE of a regression model between $E_{chiller}$ and Q_{bldg}
$(1.26 \times CV_{aux})/\sqrt{m}$	2-8%	4% (*)	Effective CV-RMSE of a regression model between E_{aux} and Q_{bldg}
$(\Delta E_{chiller}/E_{chiller})_{model,bias}$	0-6%	4%	Model bias due to external prediction of $E_{chiller}$
$(\Delta E_{aux}/E_{aux})_{model,bias}$	0-6%	3% (*)	Model bias due to external prediction of E_{aux}
R	80-100%	90%	Ratio of chiller energy use to total cooling system energy use
F	5-30%	20%	Ratio of estimated savings to baseline cooling system energy use

(*) These values have been assumed fixed while generating Figure 4.

simple linear model in T only is adopted, then $p = 2$).

Let us approximate the PI uncertainty of Equation 8 as

$$\left(\frac{\Delta Q_{bldg}}{Q_{bldg}}\right)_{model,\,precision} \cong CV_{bldg} \tag{10}$$

where CV_{bldg} represents the CV-RMSE of the building model, i.e., (RMSE/Q_{bldg}) where Q_{bldg} is the annual mean hourly value of the building secondary loads.

The expression for total uncertainty in the building loads is thus the sum of errors (1), (2), and (3) added in quadrature:

$$\left(\frac{\Delta Q_{bldg}}{Q_{bldg}}\right)^2 \cong \left(\frac{\Delta Q_{bldg}}{Q_{bldg}}\right)^2_{meas,\,bldg} + (CV_{bldg})^2_{model,\,precision} + \left(\frac{\Delta Q_{bldg}}{Q_{bldg}}\right)^2_{model,\,precision} \tag{11}$$

Uncertainty expression for $E_{chiller}$ and E_{aux}

The treatment of uncertainty in $E_{chiller}$ and E_{aux} is similar and we shall simply refer to $E_{chiller}$ in the discussion that follows in this section. Sources of error (4), (5), and (6) described above are analogous to (1), (2), and (3), which apply to building loads uncertainty. Error (4) is straightforward in that a bias fraction needs to be specified. As shown in Table 2, a typical range for electricity measurement would be from 0 to 2%.

Error (2) is the prediction error of the building load model (assuming the model to have been identified from year-long data, i.e., the model has no bias error). We shall assume, for simplicity, that $E_{chiller}$ is simply a function of Q_{bldg} (Phelan et al. 1997a, 1997b) rather than adopt a more detailed analysis where the variation in $E_{chiller}$ with condensing and evaporator temperatures is considered (Gordon and Ng 1995). In this case, the uncertainty in Q_{bldg} cannot be neglected. As stated earlier, the effect of uncertainty in regressor variables on the response variable is complex mathematically, except for the simple linear regression model (Mandel 1984). Even this treatment does not allow the associated equations to be manipulated into a form appropriate for the generalized treatment adopted in this paper.

An intuitive treatment of how to determine uncertainty in a nested regression model is proposed below. Let Q_{bldg} be the building thermal secondary loads (assumed to be the chiller evaporator load) and $E_{chiller}$ be the chiller electric power consumed. Let us assume a simple linear relation between both variables (Phelan et al. 1997a, 1997b). Then the slope of the line is the reciprocal of the COP of the chiller (which is constant due to our assumption of a simple linear relationship between both variables). The uncertainty in $E_{chiller}$ when there is no uncertainty in Q_{bldg} is illustrated in Frame (a) of Figure 3. For point A, the uncertainty is characterized by the vertical length A´-A´´ between the prediction interval (PI) bands. When there is an uncertainty in Q_{bldg}, point A can be anywhere over the range B-C shown in frame (b). The lower and upper uncertainty bands of points B and C, respectively, are B-B´´ and C-C´. Hence, the total uncertainty in point A with an uncertainty Q_{bldg} in the regressor variable will be the vertical length A´-A´´. Strictly speaking, $E_{chiller}$ and Q_{bldg} are uncorrelated, and so one should not simply add them as shown in frame (b) of Figure 3 (which was done more as a means of describing the process intuitively). It is more accurate to add them in quadrature, as follows:

$$(\Delta E_{chiller})^2_{model,\,precision} \cong (RMSE_{chiller})^2 + (\Delta Q_{bldg} \cdot \tan b)^2 \tag{12a}$$

where b is the slope of the regression line between $E_{chiller}$ and Q_{bldg}, $RMSE_{chiller}$ is the RMSE of the model between chiller electricity use and building loads at an hourly time scale.

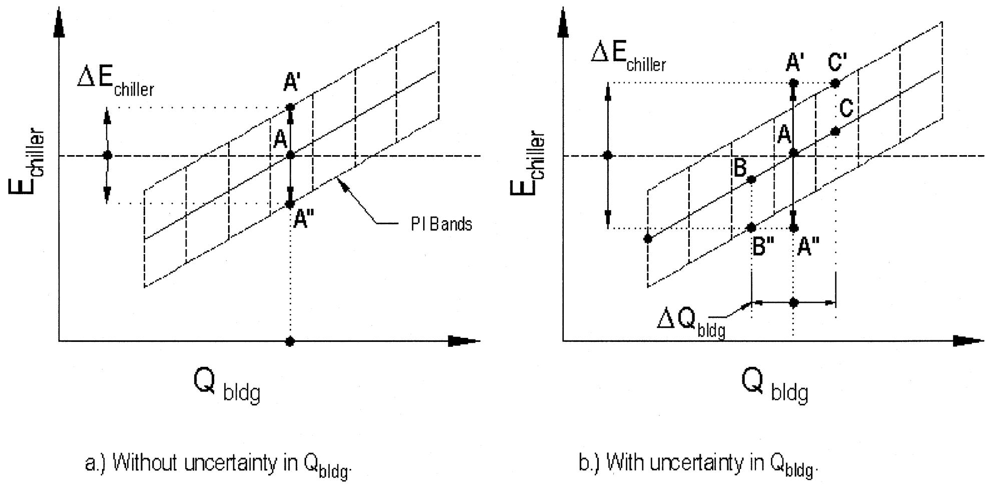

Figure 3 *Conceptual plot to illustrate how one could include the uncertainty in building secondary loads while determining uncertainty in chiller electric use assuming a simple linear relationship to exist between both variables.*

However, we are not interested in uncertainty at an hourly time scale but with the uncertainty associated with the average hourly predictions over the season or the year. Following Reddy and Claridge (1999), an approximate expression for the average of m future predictions using a regression model without uncertainty in the regressor variable is given by

$$\left(\frac{\Delta E_{chiller}}{E_{chiller}}\right)_{model,\,precision} \cong 1.26 \times \frac{CV_{chiller}}{\sqrt{m}} \tag{12b}$$

where $CV_{chiller}$ represents the CV-RMSE of the chiller model, i.e., (RMSE/$E_{chiller}$) where $E_{chiller}$ is the annual mean hourly model-predicted value of the chiller electric power. The above expression is valid for m "independent" observations, i.e., when energy use during successive hourly periods has no serial correlation. Typically serial correlations of building energy use at hourly time scales are in the range of 0.85-0.98, and so m cannot be chosen as 8760 for annual calculations. Reddy and Claridge (1999) also discuss and present an equation for calculating the number of "independent" observations in a serially correlated sample to be used in Equation 12b.

Finally, since tan $b = (E_{chiller}/Q_{bldg})$, we have from the above two equations:

$$\left(\frac{\Delta E_{chiller}}{E_{chiller}}\right)^2_{model,\,precision} = \left(\frac{1.26 \times CV_{chiller}}{\sqrt{m}}\right)^2 + \left(\frac{\Delta Q_{bldg}}{Q_{bldg}}\right)^2 \tag{12c}$$

where $(\Delta Q_{bldg} / Q_{bldg})^2$ is given by Equation 11.

By analogy to Equation 11, the expression for total uncertainty in the annual mean hourly chiller electricity use is

$$\left(\frac{\Delta E_{chiller}}{E_{chiller}}\right)^2 \cong \left(\frac{\Delta E_{elec}}{E_{elec}}\right)^2_{meas,\,bias} + \left(\frac{\Delta E_{chiller}}{E_{chiller}}\right)^2_{model,\,precision} + \left(\frac{\Delta E_{chiller}}{E_{chiller}}\right)^2_{meas,\,bias} \tag{13}$$

An analogous expression can also be derived for the auxiliary energy use.

Final Fractional Uncertainty Equations

The final expressions for the fractional uncertainty $(\Delta E/E)$ and subsequently $(\Delta\$_{save}/\$_{save})$ can now be deduced.

The total uncertainty in the annual mean hourly energy use is

$$\Delta E^2_{total} = [\Delta E^2_{aux} + \Delta E^2_{chiller}]$$

or

$$\left(\frac{\Delta E_{total}}{E_{total}}\right)^2 = \left(\frac{\Delta E_{aux}}{E_{aux}}\right)^2\left(\frac{E_{aux}}{E_{total}}\right)^2 + \left(\frac{\Delta E_{chiller}}{E_{chiller}}\right)^2\left(\frac{E_{chiller}}{E_{total}}\right)^2 \tag{14a}$$

$$= [eq.13]_{aux} \cdot (1-R)^2 + [eq.13]_{chiller} R^2 \tag{14b}$$

where R is the ratio of the mean electricity consumed by the chiller to the mean total use (i.e., chiller plus auxiliary).

The above equation can be introduced into Equation 5 to yield the final expression for $(\Delta\$_{save}/\$_{save})$.

GENERATING AND USING A NOMOGRAPH BASED ON THE ABOVE EQUATIONS

Inspection of the various equations presented in the previous sections suggests that several factors come into play while performing an engineering uncertainty analysis for a cooling system. It is difficult to acquire an intuitive understanding of the relative importance of these factors from the equations or even from the standard types of sensitivity charts where one plots the variation of the total uncertainty against each of these sources of uncertainty in turn while keeping the others constant (Coleman and Steele 1989).

In order to make the uncertainty analysis convenient to use, a graphical tool has been developed, which consists of six

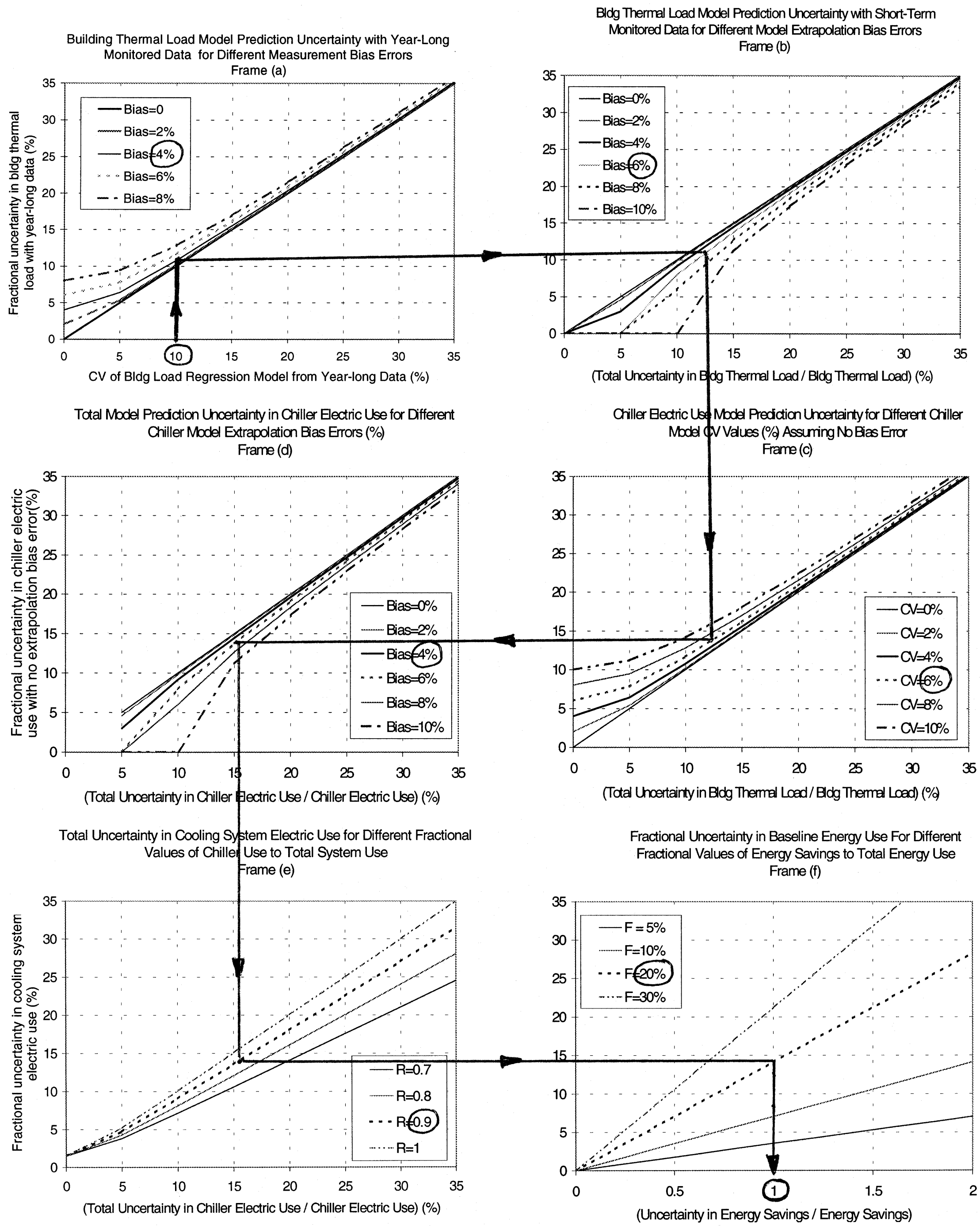

Figure 4 *Nomograph to determine the final uncertainty in energy or cost savings of a cooling system due to various individual measurement and regression model prediction uncertainties. An illustrative example as explained in the text is also shown.*

interlocked graphs plotted as shown in Figure 4. The user starts with graph (a) and moves through them to end with the final savings uncertainty value in graph (f).

Uncertainties due to measurement of Q_{bldg} along with the goodness-of-fit of the regression model between Q_{bldg} and other variables are contained in graph (a), while graph (b) captures the bias error due to model extrapolation error. Next, given that the bias uncertainty in electricity measurement of $E_{chiller}$ and E_{aux} is small, we have assumed a fixed value of 1% full scale, as indicated in Table 2, while generating graph (c). This graph includes the goodness-of-fit of the regression model between $E_{chiller}$ and Q_{bldg} while graph (d) takes in the model extrapolation bias of the chiller model. In order not to make the nomograph too tedious to use, we have not explicitly included the ability to account for the variation in uncertainties of E_{aux} either due to model internal prediction or due to model external prediction bias error. Graphs (c) and (d) assume that the regression model for E_{aux} has a fixed precision value of 4% and a fixed bias prediction error of 3%. Usually, the auxiliary energy use is much smaller than the chiller energy use, thus justifying this assumption. Graph (e) accounts for the ratio R of chiller electricity use to that of the total cooling system. Finally, graph (f) takes into account the savings fraction F_{save}, which obviously is the most influential of all parameters.

The use of this nomograph is illustrated by means of an example. The specific numerical values chosen are given in Table 2. Entering the nomograph in graph (a) with the building load model CV of 10%, proceed up vertically to the building load measurement bias error of 4%. Next, proceed horizontally into graph (b) to the intersection with the model external bias error of 6%. Under these circumstances, the fractional uncertainty in building loads denoted by $(\Delta Q_{bldg}/Q_{bldg})$ (see Equation 11) can be read off as 13%. Continue descending vertically into graph (c) until the chiller model value, i.e., = 6%, line is reached. Next one goes horizontally into graph (d) to the chiller model external prediction line of 4%. Reading downward, we note that the total uncertainty in the mean annual electricity use of the chiller is 16%. Proceed downward to the line representing R = 0.9 in graph (e) and finally move horizontally into graph (f) where we have selected F = 20%. The x value of this intersection provides the final fractional uncertainty in the savings estimate, namely, $(\Delta\$_{save}/\$_{save})$ following Equation 5. We note that the fraction is 1, denoting 100% uncertainty in our savings estimate although the uncertainty in our chiller electricity use is only 16%.

CONCLUDING REMARKS

This paper discussed the various sources of uncertainty that need to be treated while ascertaining the uncertainty in the energy and cost savings resulting from an ECM project. A methodology, along with pertinent equations for performing the uncertainty analysis, was presented, and, based on realistic ranges of variation of the various parameters, an uncertainty nomograph consisting of six interlocked graphs was generated. This nomograph can be used to quickly ascertain the uncertainty in the savings estimate for a particular application or, alternatively, to determine the accuracy required at each level of analysis to attain a desired overall uncertainty in the calculation of energy savings. Thus, the nomograph can be used for actual projects or for project planning. Inspection of the nomograph clearly indicates that the fraction F (i.e., savings to baseline energy use) has the most influence on the savings uncertainty fraction. So it would be unrealistic to expect a low uncertainty in savings when F itself is low.

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NOMENCLATURE

C	=	unit cost of electricity
E	=	energy use, baseline energy use
F	=	ratio of the energy savings to baseline energy use
m	=	number of post-ECM observation points
n	=	number of pre-ECM observation points
p	=	number of model parameters (= k + 1)
Q	=	thermal loads
T	=	dry-bulb temperature
R	=	ratio of mean chiller electricity use to that of the plant (chiller plus auxiliary)
t	=	t-statistic
$\bar{X}$	=	mean value of X
$\hat{X}$	=	model-predicted value of X
CV	=	coefficient of variation
ΔE	=	uncertainty in E
RMSE	=	root mean square error

Subscripts

aux	=	auxiliary (fans, pumps, cooling tower)
bldg	=	building
chiller	=	chiller
elec	=	electricity
meas	=	measurement
o	=	outdoor
post	=	post-ECM
pre	=	pre-ECM or baseline
save	=	savings

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