Prediction Uncertainty of Linear Building Energy Use Models With Autocorrelated Residuals

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Autocorrelated residuals from regression models of building energy use present problems when attempting to estimate retrofit energy savings and the uncertainty of the savings. This paper discusses the causes of autocorrelation in energy use models and proposes a method to deal with autocorrelation. A hybrid of ordinary least squares (OLS) and autoregressive (AR) models is developed to accurately predict energy use and give reasonable uncertainty estimates. Only linear models are considered because both the data and the physical theory for many commercial buildings support this choice (Kissock, 1993). A procedure for model selection is presented and tested on data from three commercial buildings participating in the Texas LoanSTAR program. In every case examined, the hybrid OLS-AR model provided the best estimate of energy use and the most robust estimate of uncertainty.

Introduction

A crucial requirement in being able to promote and sustain energy conservation measures in commercial buildings is the ability to perform careful and reliable appraisals of exactly how much energy has been saved. Because of the potentially large discrepancy between predicted and actual savings, many programs such as the Texas LoanSTAR program (Claridge et al., 1991) require that energy savings due to the retrofit be measured. This requires that data acquisition equipment to monitor building energy use be installed in a building for a suitable period before the retrofits are carried out and that the equipment remain in the building, possibly throughout the life of the retrofit. Varying weather conditions (and other variables such as internal loads and schedules) between the pre-retrofit and post-retrofit periods can influence energy use and may obscure the change in energy use caused by the retrofit. A more accurate measure of retrofit savings is provided by developing a weather-dependent model of the building's pre-retrofit energy use and then using this model to predict the building's pre-retrofit energy use under post-retrofit conditions. The difference between these simulated or predicted values and the actual measured energy use can provide a sound estimate of the retrofit savings. Such an approach has been adopted in the LoanSTAR program (Kissock et al., 1992a) with the model approach being in most cases statistical in nature and involving a regression model at the daily time scale. This study has been initiated in the framework of the LoanSTAR program with the specific objectives of reevaluation and refining previous work (e.g., Kissock et al., 1992b; Reddy et al., 1992) and developing more accurate and robust model identification procedures along with statistically sound methods of determining the uncertainty of the savings. In short, this paper:

- discusses possible causes of autocorrelation in energy use models;
- develops a hybrid of OLS and autoregressive (AR) energy models to accurately predict energy use and give reasonable uncertainty measures;
- outlines a procedure for identifying the most appropriate linear model when autocorrelation is present; and
- applies this procedure to several real data sets, and validates the models' predictive ability and estimated uncertainty.

Autocorrelation in Energy Models

When data are fit using a regression model, the errors may not be independent of each other across time; in such a situation the errors are said to be non-random or autocorrelated. The presence of autocorrelation is important because it will cause statistical problems. Most importantly for our purposes, estimated prediction error bounds will be too small, leading to undue confidence being placed in the accuracy of predicted energy use. In addition, the mean square error of the regression fit may underestimate model variance, and the standard errors of the OLS regression coefficients will be too small (Neter, Wasserman and Kutner, 1990). This effect can be explained in a simple intuitive manner. If sampling is done from a population such that the data points are correlated, this effectively reduces the number of "independent" sample data drawn from the population. This, in turn, increases the uncertainty interval lengths, which are inversely related to the number of independent data points.

Autocorrelation of errors from OLS models of building energy data has been noted in LoanSTAR and other buildings (Verdi, 1989; Reddy et al., 1992). Autocorrelation may be caused by time dependent operational changes in the building or by the omission of variables that may influence energy use, such as humidity, occupancy loads and solar radiation. By operational changes we mean HVAC system changes such as resetting the hot-deck temperature or switching chilled water pumps on or off. Operational changes such as these are often more easily modeled by grouping the data in a time series fashion according to similar modes of operation, rather than by attempting to include the appropriate physical variables in the model. For example, if the supply of steam to a building is shut off during the summer months, an indicator model that separates steam use in the summer from steam use during the remainder of the year is warranted. Less significant operational changes may also occur on a regular basis. These unknown or unmeasured changes are difficult to model and when not accounted for can become additional sources of autocorrelation in energy use models.

The other major cause of autocorrelation is the omission from the model of important influencing variables (Neter, Wasserman and Kutner, 1990). Outdoor air temperature, a readily available

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measurement, has been shown in several studies (e.g., Fels, 1986; MacDonald and Wasserman, 1988; Kissock et al., 1992a) to be an important predictor of heating and cooling energy use. However, many of the other variables which are believed to be important drivers of building energy use are inherently difficult or expensive to measure with the precision and robustness necessary for improving the predictive ability of a model. For example, high levels of ambient humidity increase a building's cooling load, but accurate humidity data are difficult to obtain. Internal loads, such as those generated by human activity and electrical equipment, are also important components of a building's space cooling load, but are difficult to measure. In cases like these, the omission of important predictor variables from the model and the consequent autocorrelation of model residuals may be unavoidable.

If possible, autocorrelation should be addressed by redesigning the model to include time dependent changes and all significant predictor variables. Model redesign is one remedy for autocorrelation, and has been explored in previous work (Ruch, Kissock, Reddy, 1993). As noted above, however, practical measurement constraints often limit the effectiveness of model redesign. If the autocorrelation cannot be completely removed by model redesign, another modeling approach is to use an autoregressive model to eliminate the autocorrelation of errors (Verdi, 1989; Reddy et al., 1992). Autoregressive models of daily energy use perform best when the energy use from the previous day(s) is available to be used in a corrective error term. However, an AR model with a corrective error term is not appropriate for estimating savings from the post-retrofit period because the building's pre-retrofit energy use, which is an essential component of the corrective error term, is no longer available. In some cases, an AR model without a correcting error term may adequately predict energy use; however, in all of the LoanSTAR cases so far examined, AR models without corrective error terms have not predicted energy use as well as standard OLS models. Thus, standard AR models have limited use when predicting savings.

In this paper we propose a "hybrid" predictor for predicting pre-retrofit energy consumption several days into the post-retrofit period. The hybrid predictor uses the OLS regression coefficients, but estimates the model variance and prediction error differently, in a manner designed to account for the autocorrelation. The hybrid predictor benefits from both the superior prediction accuracy of OLS regression coefficients and the more accurate estimate of uncertainty associated with AR models. Our focus is on the use of these regression models, rather than, say, neural network models, because the prediction uncertainty of these regression models can be statistically estimated.

Verdi (1989) and Reddy et al. (1992) have investigated means of redesigning OLS energy use models by using autoregressive models to eliminate or reduce autocorrelation. Based on these studies, we assume first order autocorrelation of errors for statistical simplicity and because of significantly improved data fits.

A simple linear OLS model of predicted energy use, $\hat{E}_k$, of day k is

$$\hat{E}_k = \alpha + \beta T_k, \tag{1}$$

where T_k denotes the temperature on day k.

If autocorrelation is present, an autoregressive model (assuming first order autocorrelation) of the energy use has two components: a structural term and a conditional error term. The predicted energy, $\hat{E}_k$, of day k is

$$\hat{E}_k = a + bT_k + \rho\epsilon_{k-1} \tag{2}$$

where T_k is the temperature on day k, ρ is the correlation coefficient, and $\epsilon_{k-1} = E_{k-1} - \hat{E}_{k-1}$ is the prediction error from the $k - 1$st day. The structural component of the model is

$$a + bT_k$$

while the conditional error component is

$$\rho\epsilon_{k-1}.$$

Usually the regression coefficients of the OLS model (α and β) will be different than those of the autoregressive model (a and b). In many applications, there is little difference in the models' coefficients but a significant difference in their error diagnostics (Neter, Wasserman and Kutner, 1990); the autoregressive model will predict much like OLS but with more reliable error bounds. In such cases, the conditional error component is only a minor contributor to the data fit.

On the other hand, there are also cases where the conditional error component is the major contributor to the data fit, and the structural component alone is a poor predictor (SAS/ETS 1993, p. 208). Unfortunately, the prediction of pre-retrofit energy consumption under post-retrofit weather conditions cannot use updated daily errors since measured pre-retrofit energy data from post-retrofit weather conditions is not available. Consequently the conditional error component cannot be used in our predictor model. Therefore the prediction is done essentially by the structural component of the autoregressive model. Hence the autoregressive model will not be appropriate if the component alone is a poor predictor. This was the case for the case study buildings discussed below, so a hybrid model, discussed in the next section, was developed.

A Hybrid Predictor With Reliable Uncertainty Estimates

For many buildings it may not be possible to completely eliminate autocorrelation through model redesign, and autoregressive models may be inappropriate for the reasons discussed above. Fortunately, even if autocorrelation is present, the regression coefficient estimates given by OLS are reasonable in the sense of being statistically unbiased (Theil, 1971, p. 254), and thus give the best estimates of the coefficients under the circumstances. In this situation, the problem with an OLS model is that the usual error diagnostics are biased and may severely underestimate the prediction uncertainty. We therefore propose to use a "hybrid" predictor, using the OLS regression coefficients plus a term that accounts for the known autocorrelation. Most importantly, this hybrid approach also accounts for the autocorrelation in its uncertainty estimate.

We split the data into "pre" and "post" portions. The "pre" portion is used to develop the model and the "post" portion is used to test the model's predictive ability. To develop this hybrid predictor, we first note that because of the autocorrelation, which we assume to be first order, the errors from the OLS fit on consecutive days k and $k - 1$ are related as

$$\epsilon_k = \rho\epsilon_{k-1} + \delta_k \tag{3}$$

where δ_k is a random error term, and ρ is the autocorrelation coefficient of the residuals. Thus, to best predict the energy use E_1 on the first day of the "post" period we use Equation 3 to obtain

$$E_1 = a + bT_1 + \epsilon_1 = a + bT_1 + \rho\epsilon_0 + \delta_1. \tag{4}$$

where ϵ_0 is the actual error on the final day of the "pre" period. Because E_1 and δ_1 are unknown, our best predictor of E_1 is

$$\hat{E}_1 = a + bT_1 + \rho\epsilon_0. \tag{5}$$

Similarly, our best predictor of E_2 is

$$\hat{E}_2 = a + bT_2 + \rho^2\epsilon_0 \approx a + bT_2 + \rho\epsilon_1. \tag{6}$$

By induction, it follows that the best predictor of day k of the "post" period is

$$\hat{E}_k = a + bT_k + \rho^k \epsilon_0. \quad (7)$$

We shall refer to Eq. 7 as a hybrid predictor, because the regression coefficients are computed using OLS, and the term $\rho^k\epsilon_0$ estimates the autocorrelation effect. Since where $\vec{1}$ is a column of ones. Note that matrix $P\Psi P^{-1} - 2PV + \Psi_*$ in Eq. 11 is $m \times m$, the length of the "post" period. Pre and post-multiplying of this matrix by $\vec{1}$ is just a compact notation for summing all of the terms in the matrix.

The actual computation of this estimate of the variance of the total prediction error involves a straightforward matrix calculation, which can be done easily using matrix oriented software (SAS/IML is excellent for this task). The variance can then be used to obtain prediction error bounds as follows. At a confidence level $1 - \alpha$, the prediction error bound is

$$t_{\alpha/2}\, \sigma \sqrt{\vec{1}' \cdot (P\Psi P^{-1} - 2PV + \Psi_*) \cdot \vec{1}} \quad (12)$$

where $t_{\alpha/2}$ is the t-statistic with $(n - k)$ degrees of freedom where n is the number of pre-retrofit observations and k is the number of regression parameters in the model (see Ruch, 1992 for more details). Thus, the total prediction error $\sum_{i=1}^{m} (\hat{E}_* - E_*)_i$ for the "post" period energy use is expected to be less than its prediction error bound at the specified confidence level.

There are two major sources of variance when the "post" data set is fairly long (two or more months). In this situation, the correlation between pre-retrofit and post-retrofit disturbances becomes small when compared to the other sources of error. We can thus approximate this condition by setting $\sigma^2 V$, the matrix of covariances between pre-retrofit and post-retrofit disturbances, equal to zero in the definition of matrix P and in Eq. 11. The major sources of variance in the sum of daily prediction errors are therefore: (a) the matrix $\sigma^2 P\Psi P'$, which is due to sampling error in the estimation of the regression coefficients; and (b) the matrix $\sigma^2\Psi_*$, which is due to variance in the post-retrofit disturbances.

A problem related to finding prediction error bounds is estimating the model error terms' variance σ^2. Unfortunately, the standard OLS estimate of σ^2 by the mean square error (MSE) of the regression is biased (Theil, 1971, p. 256). This requires that an alternate estimate of σ^2 be made. An unbiased estimate of the variance σ^2 under the assumption of first order autocorrelation is (Theil 1971, p. 256)

$$\hat{\sigma}^2 = \frac{RSS}{\sum_{k=1}^{n} (M\Psi)_{kk}} \quad (13)$$

where RSS is the residual sum of squares from the OLS regression. This will generally be larger than the MSE, which partly explains why the OLS error bounds are too small. The $|\rho| < 1$, this term rapidly diminishes with time and is negligible after about 30 days into the "post" period, even when $|\rho|$ is close to 1.

To outline the derivation of the proper error estimates, we now switch to matrix notation. Full details of the derivation are available in Ruch (1992). The model above can be written as $E = X\beta + \epsilon$, where E is a $(n \times 1)$ vector of pre-retrofit energy observations and X is a $(n \times k)$ matrix whose first column is a column of ones and remaining columns are the pre-retrofit observations of the independent variables (such as temperature, humidity or indicator variables).

To describe m observations of post-retrofit energy use, E_*, we have $E_* = X_*\beta + \epsilon_*$ where X_* is the matrix of post-retrofit independent variables. Let $\sigma^2\Psi$ and $\sigma^2\Psi_*$ be the covariances of pre-retrofit and post-retrofit disturbances respectively, and let $\sigma^2 V$ be the covariances *between* pre- and post-retrofit disturbances where σ^2 is the variance of the model error terms and first order autocorrelation is assumed. Then the optimal hybrid predictor (Eq. 7) can be written in matrix notation as:

$$\hat{E}_* = X_*\hat{\beta} + V'\Psi^{-1}(E - X\hat{\beta}) \quad (8)$$

where $\hat{\beta} = (X'X)^{-1}X'E$ is the usual OLS estimator of the regression coefficients. The prediction error is

$$\begin{aligned}\hat{E}_* - E_* &= X_*(\hat{\beta} - \beta) + V'\Psi^{-1}(E - X\hat{\beta}) - \epsilon_* \\ &= X_*(X'X)^{-1}X'\epsilon + V'\Psi^{-1}M\epsilon - \epsilon_* \\ &= P\epsilon - \epsilon_* \end{aligned} \quad (9)$$

where $M = I - X(X'X)^{-1}X'$ and $P = X_*(X'X)^{-1}X' + V'\Psi^{-1}M$. The variance-covariance matrix of the prediction error is thus

$$\text{var}(\hat{E}_* - E_*) = \sigma^2(P\Psi P' - PV - V'P' + \Psi_*) \quad (10)$$

The variance of the prediction error for day i is the $i - i$ element of this $m \times m$ matrix, and the off-diagonal elements are the covariances of the daily prediction errors. So the variance of the sum of the daily prediction errors is

$$\begin{aligned}\text{var}\left(\sum_{i=1}^{m} (\hat{E}_* - E_*)_i\right) &= \vec{1}' \cdot \text{var}(\hat{E} - E_*) \cdot \vec{1} \\ &= \sigma^2 \vec{1}' \cdot (P\Psi P^{-1} - 2PV + \Psi_*) \cdot \vec{1} \end{aligned} \quad (11)$$

variance estimate described in Eq. 13 is used for estimating the hybrid model's variance.

If there is no autocorrelation present in the original OLS fit, these results still hold and become much simpler since the matrix V is now zero, and the matrices Y and Y^{-1} become the identity matrix. In this case the estimate of the variance of the prediction error is (Theil, 1971, p. 123):

$$\text{var}\left(\sum_{i=1}^{m} (\hat{E}_* - E_*)_i\right) = \sigma^2 \vec{1}' \cdot (X_*(X'X)^{-1}X'_* + I) \cdot \vec{1}, \quad (14)$$

which is used for computing prediction error bounds for OLS models.

A common but incorrect practice for computing prediction error bounds of the sum of daily predictions is to sum in quadrature the prediction error bounds of the daily predictions (Kissock et al., 1992b; Reddy et al., 1992). This amounts to assuming that the daily prediction errors are independent:

$$\text{var}\left(\sum_{i=1}^{m} (\hat{E}_* - E_*)_i\right) = \sum_{i=1}^{m} \text{var}(\hat{E}_* - E_*)_i \quad (15)$$

or, equivalently, that the off-diagonal entries in matrix 10 are all zero. However, this is not generally the case because all of the daily predictions of energy use are based on the same estimated regression coefficients and are therefore correlated. Consequently, summing in quadrature will underestimate the correct prediction error bound. This is true even when there is no autocorrelation of errors in the OLS regression fit (Theil, 1971, p. 122).

Procedure for Linear Model Fitting When Autocorrelation is Present

A variety of techniques for explaining and dealing with autocorrelation in modeling energy use have been discussed above. In this section we develop a procedure for putting these ideas together. We assume that the given building data suggest that energy is a linear function of temperature, i.e., no temperature change-point is apparent that requires PRISM (Fels, 1986) or a four-parameter fit (Ruch and Claridge, 1991).

The first step is to perform an OLS fit to the data. If there is no indication of autocorrelation or other statistical problems, use the OLS model for predicting energy use and Eq. 14 for estimating the prediction error bounds. If autocorrelation exists, redesign the model if possible, looking for time-dependent operational changes or omitted variables. If, after this is done, there

Table 1 **Descriptions of buildings used in case studies. CW is cooling energy and HW is heating energy. Classroom building data from 5/1/91 through 10/1/91 were not available due to metering difficulties.**

Building	Principle use	Type of data	Dates of "pre" data	Dates of "post" data
Engineering Center	Classrooms, offices, laboratories	CW (GJ/day) HW (GJ/day)	1/15/90–6/30/90 6/1/90–10/31/90	7/1/90–11/27/90 11/1/90–11/27/90
Campus Library	Library, offices, computer facility	CW (GJ/day)	8/7/91–1/31/92	2/1/92–4/30/92
Classroom Building	Classrooms, offices	HW (GJ/day)	12/5/90–4/30/91	11/1/91–3/16/92

is still autocorrelation present, or if redesigning is not possible, try fitting an autoregressive model. If the structural component is the major contributor—i.e., its R^2 is nearly as high as the OLS fit and its fit to the data reasonable—then this autoregressive model can be used as a predictor. Note that this has not been the case for any LoanSTAR buildings studied thus far. Finally, after all possible model redesign has been done, and if the autoregressive model is inappropriate, use the hybrid model (Eq. 7) to predict energy use.

Model Comparison and Validation

In this section we test the performance of various models on daily energy use data which are split into "pre" and "post" portions in order to simulate the prediction of pre-retrofit energy use during post-retrofit weather conditions. The models' uncertainty estimates and predictive abilities are then compared.

Hybrid and OLS Models Compared on Several Buildings. We first compare the performance of the hybrid and OLS models on real energy use data from three buildings participating in the LoanSTAR program. Data sets of cooling and heating energy use at a large engineering center, cooling energy use at a campus library and heating energy use at a classroom building were selected. In all cases, only outdoor temperature was available as an independent variable and energy appeared to be a linear function of temperature. For each building, the data were split into two parts: hypothetical "pre" and "post" periods with energy use modeling done on the "pre" data and the models tested on the "post" data. The "pre" set was selected so that the range of daily temperatures was as representative as possible of the annual temperature variation for the region. Given this constraint, the data were split in half where possible. The data periods are given in Table 1.

The modeling procedure discussed above led to a hybrid predictor of the form given by Eq. 7 for each building. For comparative purposes, both OLS and hybrid models were fit to each building's "pre" period data set. The models were then used to predict the cumulative energy use for the "post" period, and prediction error bounds were calculated at the 95% and 99% confidence levels. To illustrate the problem with summing the prediction error bounds in quadrature, prediction error bounds were also computed in this way. The results are listed in Table 2.

Note that in every case the hybrid model's prediction error bound was greater than that of the OLS model. In all four cases the OLS error is greater than the predicted OLS error bound at the 99% confidence level, while the hybrid error is less than its predicted error bound at the 95% level. This supports the theoretical argument that OLS prediction error bounds are generally too small and hybrid error bounds are more realistic. The quadrature method produced prediction error bounds even smaller than the OLS error bounds for every case, again supporting the theoretical argument that summing errors in quadrature underestimates the true error.

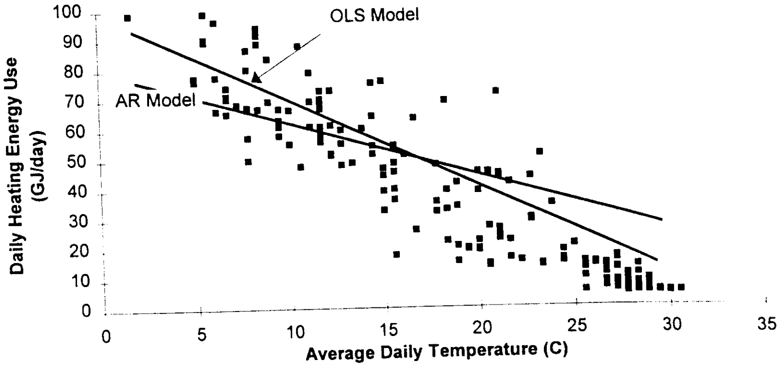
The hybrid method compares favorably with OLS in its predictive ability, since its prediction of heating energy for the engineering center, where the "post" period is short (27 days), is considerably more accurate than OLS. The two methods have essentially the same prediction accuracy in the other three cases.

Model Redesign and Energy Prediction: A Case Study Building. This case study discusses both model redesign and the hybrid predictor. The building examined is the campus library described in Table 1. In this example, daily temperature and heating energy use data from August 1, 1991 through April 30, 1992 were analyzed. The data were split at the end of January 1992 into hypothetical "pre" and "post" portions. The "pre" portion was used to develop the model and the "post" portion was used to test the model's predictive ability. A plot of the "pre" energy use versus temperature suggests that heating energy use is a reasonably linear function of temperature (Fig. 1*a*). An OLS fit to the "pre" data yielded an R^2 value of .90, but the residuals were significantly correlated over time, with an autocorrelation coefficient of .90. A full autoregressive model was fit to the data, which improved the fit greatly (R^2 = 0.97), but could not be used as a predictor since the error component was not available during the "post" period. The structural component of the model alone was clearly skewed (Fig. 1*a*) and had a poor R^2 value of 0.59. It was therefore unlikely to predict properly without the aid of the conditional error component of the model and so was rejected as a predictor.

A closer analysis of the data revealed three distinct patterns of energy use: the first prior to November 3, the second during a 14 day period over the Christmas and Martin Luther King holidays, and a third period covering the rest of the "pre"

Table 2 **Predictive ability and uncertainty of OLS and Hybrid models compared to actual data. The percentages are relative to the actual energy use. For a correct estimate of uncertainty, the actual prediction error should be less than the prediction error bound at the given confidence level.**

Building and Energy Type	Model Type	Autocorrelation coefficient	Predicted energy use (GJ)	Actual energy use (GJ)	Actual prediction error	Prediction error bound at 95% confidence level	Prediction error bound at 99% confidence level
Library	OLS	.78	8419	7779	8%	3%	4%
Heating	Hybrid	.78	8400	7779	8%	8%	11%
Eng. Center	OLS	.96	1268	1569	19%	7%	9%
Heating	Hybrid	.96	1393	1569	11%	33%	44%
Eng. Center	OLS	.73	3244	3435	6%	4%	5%
Heating	Hybrid	.73	3243	3435	6%	9%	12%
Class Room	OLS	.94	2194	2609	16%	3%	4%
Heating	Hybrid	.94	2194	2609	16%	17%	22%



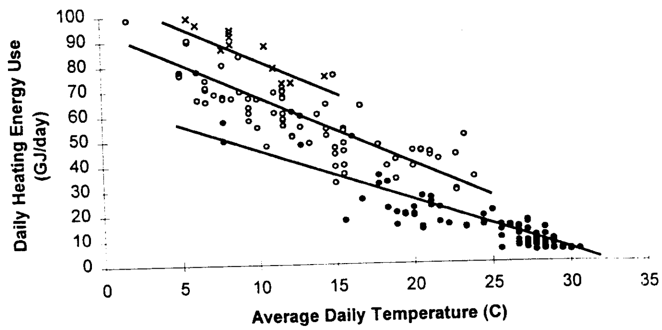


Fig. 1 Daily energy use during the "pre" period at the campus library: (*a*) energy use with OLS model and the structural part of the AR model; (*b*) energy use by mode of operation with hybrid indicator models. In (*a*) the AR model is clearly skewed. In (*b*) the hybrid indicator models are the best predictors of energy use.

period (Fig. 1*b*). Inquiries with the building operators confirmed that there were indeed three modes of operation:

1) from August, 1991 through October, 1991 steam used for interior zone reheat is turned off;
2) from November, 1991 through January, 1992 steam used for interior zone reheat is turned on; and
3) operating hours were severely reduced during the holidays.

Following our procedure, three predictor models were developed on the "pre" data and tested on the "post" data. The first was the OLS model mentioned above. The second was an "indicator model" that took into account all three operating stages using indicator variables, but did not allow for the remaining autocorrelation in its uncertainty estimate. A third "comprehensive" model took into account all three operating stages as well as the autocorrelation. The results are listed in Table 3 and are discussed below.

The simple OLS model of energy use during the "pre" period is

$$E_k = 99.27 - 3.23T_k \qquad (16)$$

When this model is used to predict energy use in the "post" period, the prediction is 17% too high. In addition, its prediction error is substantially greater than the OLS prediction error bound at the 95% confidence level (see Table 3). This severe underestimation of uncertainty is typical of OLS models when autocorrelation is ignored. It is also clear from Fig. 1 that the slope of the OLS model line is steeper than the slope of any of the model lines during the three operating modes. If one is interested in the rate of change of energy use with respect to temperature, the OLS model will give false information. In this case, the slope given by the original autoregressive fit, although skewed and not acceptable as a predictor of overall energy use, is considerably closer to the slopes for the three operational modes. Also note that the *RMSE* estimated by OLS is 8.6, but the unbiased estimate using Eq. 13 is higher—about 8.9—which is part of the reason the prediction uncertainty is too small.

An autoregressive model with an error correction term improved the fit slightly. However, when only its structural component was considered, the fit was worse (Fig. 1*a*), with an R^2 of .82, and so this autoregressive model was ruled out as a predictor.

Using indicator variables for the three time periods, an indicator model incorporating all three operational modes was devel-

Table 3 Comparison of models' predictive ability and uncertainty estimates using campus library heating energy use data. The percentages are relative to the actual energy use. Redesigning the OLS model improves the prediction accuracy and error bound.

Model type	R^2	Autocorrelation coefficient	Predicted energy use (GJ)	Actual energy use (GJ)	Actual prediction error	Prediction error bound at the 95% confidence level
OLS	0.90	0.94	3725	3184	17%	6%
Indicator	0.96	0.71	3161	3184	1%	5%
Comprehensive	0.96	0.71	3162	3184	1%	11%

oped (see Ruch, Kissock, Reddy, 1993 or Neter, Wasserman, Kutner 1990 for details on indicator models). This increased the R^2 to .96 and lowered the autocorrelation coefficient to .71, justifying the new model (Fig. 1*b*). Its equation is

$$E_k = 94.70 - 2.78 I_1 T_k - 27.73 I_2 - 2.09 I_2 T_k + 15.59 I_3 \quad (17)$$

where

$$I_1 = \begin{cases} 1 & \text{during } 11/4/91-1/31/92 \\ 0 & \text{otherwise} \end{cases},$$

$$I_2 = \begin{cases} 1 & \text{during } 8/1/91-11/3/91 \\ 0 & \text{otherwise} \end{cases},$$

and

$$I_3 = \begin{cases} 1 & \text{during } 12/21/91-1/1/92, \\ & 1/19/92,\ 1/20/92\ . \\ 0 & \text{otherwise} \end{cases}$$

Although this model predicted well, the autocorrelation was still significant and thus this model was not optimal.

Finally, a third "comprehensive" hybrid predictor was developed using the OLS regression coefficients from the indicator model (Eq. 17) plus a term estimating the autocorrelation effect. The equation for this model is

$$E_k = 94.70 - 2.78 I_1 T_k - 27.73 I_2 - 2.09 I_2 T_k + 15.59 I_3 + (0.71)^k \epsilon_0 \quad (18)$$

where $\epsilon_0 = 0.12$ is the actual error on the final day of the "pre" period and the other variables are the same as defined above.

Taking March 1, 1992 as the date when steam for interior zones was once again turned off, these predictors were applied to the "post" data. Both were significantly more accurate than the OLS model, with actual prediction errors of approximately 1% (see Table 3). The actual prediction errors for both the indicator and comprehensive models (1% for each model) were less than their respective prediction error bounds (5% and 11%), indicating that both models gave satisfactory uncertainty estimates.

Conclusions and Future Directions

Attention to autocorrelation can improve the modeling process in two ways. First, its very existence suggests that the model may be improved, either by including omitted variables if possible, or by redesigning the model using indicator variables to account for different time-dependent modes of operation. Secondly, superior estimates of prediction uncertainty can be made using a "hybrid" approach that accounts for autocorrelation if the model cannot be redesigned to eliminate autocorrelation. These two points have been developed and illustrated on case study buildings where energy use was a linear function of temperature.

This work suggests several areas of future research. First, the study of linear models with autocorrelation should be followed up with a similar analysis of non-linear models such as PRISM (Fels, 1986) and four-parameter change-point models (Ruch and Claridge, 1991). Secondly, relating this prediction analysis to NAC (Normalized Annual Consumption) would be worthwhile, considering the importance of NAC in energy analysis (Reynolds et al., 1990; Ruch and Claridge, 1992). It is intuitively plausible that an estimate of "post" energy use, divided by the length of the "post" period, should asymptotically approach the model's NAC estimate as the "post" period lengthens. Work is being done to prove this conjecture, along with results relating prediction uncertainty and the standard error of a NAC estimate. Third, it can be seen from Table 3 that the indicator model's prediction error bound is adequate for this case study. The theory suggests that in general the comprehensive model's prediction error bound is more appropriate. However, it would be worth more study to determine whether this phenomenon occurs in other case studies. Finally, the different time-dependent modes of operation for the case studies in this study were found using ad hoc data and residual analysis. It would be useful to develop more systematic and rigorous methods for finding these different time-periods.

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