Information Content of Incoming Data During Field Monitoring: Application to Chiller Modeling (RP-1139)

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Monitoring and verification (M&V) of energy savings in installed systems as well as non-intrusive automated model-based fault detection and diagnosis (FDD) of HVAC&R systems both rely on the ability to identify model parameter estimates from monitored field data using either off-line or on-line techniques. Most HVAC&R systems operate in a fairly repetitive manner from day to day, while being subject to a long-term annual variation due to climatic changes and the manner in which the systems are designed to respond to them. Consequently, the benefit of collecting field data in order to identify a performance model of the system or equipment is affected by both of the two above considerations, with the benefit generally decreasing over time. We demonstrate, using monitored field data from one chiller, that the presence of a strong temporal correlation in the incoming data, along with ill-conditioning of the regressor matrix of a specific chiller model, can result in a sequence of about 300 field-monitored data points essentially having the same information content as 20 “independent” data points.

We provide a brief discussion of notions relating to “information content” of data in various disciplines and how to evaluate whether a new datum is providing additional information to an already available data set. Subsequently, we select and discuss two mathematical definitions of information content relevant to our specific application and apply these to two field-operated chiller data sets differing both in time scale of data collection (15-minute and 1-hour) as well as duration (14 days and 5 months) in the framework of three different linear chiller models. We specifically discuss: (a) how they can provide insight into the initial length of data set needed to initiate adaptive on-line model training, (b) how they can be used to determine when the monitored data do not provide any new information likely to modify the parameter estimates of the linear models, and (c) how this initial data length depends on the model used. Though the results of the analyses presented in this paper are specific to the data sets used, the underlying notions and concepts can be of practical relevance as to how experimental measurements can be digested toward ascertaining statistically meaningful information. This is important in view of the increasing number of non-intrusive field monitoring projects currently being performed by the HVAC&R community in the context of M&V and FDD.

INTRODUCTION

Recent advances in sensor technology, data-gathering hardware, communication, data analysis, and modeling have led to dramatic strides being made in the following three areas:

a. field measurement and verification (M&V) of energy and cost savings to ensure that the installed systems meet the specified performance predictions resulting from installing energy conservation measures (ECMs);
b. automated fault detection and diagnosis (FDD) of HVAC&R systems for maintaining optimal performance via predictive maintenance; and
c. integrated automation and control of building systems and services that are meant to assist in proper facility management, which include energy management, comfort monitoring, facility operation, and services billing and communication with the energy supplier.

The scope of this paper is limited to the first two areas only. An increasing number of energy performance contracts require verification by actual field monitoring of the energy and cost savings resulting from implementing energy efficiency projects. The National Association of Energy Service Contractors developed protocols for the measurement of retrofit savings in 1992, which were followed by federal protocols, such as FEMP (1996), IPMVP (1997), and ARI (1998), and, finally, ASHRAE Guideline 14 (ASHRAE 2002). There are also numerous refereed publications in this area, for example, the ASME Special Issue (Claridge 1998) or references listed in Reddy and Claridge (2000).

Investigators and service companies are being required to develop custom measurement plans and analytical procedures for each project, which increases total project costs. An important issue during the M&V process is the duration over which pre-retrofit measurements need to be taken in order to identify a baseline performance model that accurately captures system behavior over the entire year. Obviously one would like to gather and analyze data for as short a period as possible (both during the pre-retrofit and post-retrofit periods), while meeting the verification requirements. On the other hand, HVAC&R systems are strongly influenced by both diurnal cyclic operation as well as seasonal variation in operation (for example, control setpoints such as cold deck temperatures of an air handler) and driving parameters (such as outdoor temperature), which would suggest that at least a whole year be used for model development even though a large fraction of such data may be superfluous. Essentially two types of options are available: (a) interrupted monitoring, where one would monitor, say, over a week during each month of the year so as to capture annual variability (this may, however, not be a practical option), and (b) continuous monitoring, where the monitoring is done as a block over a certain period of the year. Several studies (Kissock et al. 1998; Katipamula et al. 1998; Reddy et al. 1998; Reddy et al. 2002) have investigated the latter option in an empirical manner and made recommendations as to the season (or time of the year) that is likely to yield performance models of HVAC&R systems that provide most accurate predictions of annual performance. Though the recommendations are consistent with our physical understanding, these are anecdotal and lack a clear scientific basis as well as the means of ascertaining whether, and the extent to which, incremental monitoring and the data thus collected provide “added value or new information” to the monitored data set already obtained.

In the last few decades, the issue of fault detection and diagnosis (FDD) in the performance of engineering systems has drawn the attention of a number of researchers. With many engineering systems growing larger and more complex, the need to operate and control them safely and reliably has extended beyond the normally accepted safety-critical systems to being able to continuously operate them in an optimal manner (Chen and Patton 1999). As early as the 1960s, it was realized that faults in critical systems, such as nuclear power plants, space exploration, and weapon systems could have grave consequences. Even a minor malfunction may cause the failure of the whole system, resulting in loss of time, money, and even life. Such considerations led to research into FDD supervisory systems in order to identify even relatively minor malfunctions as early as possible, while emphasizing detection speed, sensitivity, and false alarm rate. Several textbooks have been written on this topic (for example, Himmelblau [1978], Tzafestas et al. [1987], Pouliiezos and Stravrakakis [1994], and Gertler [1998]). This optimum performance of the system would involve reducing the occurrence of sudden, disruptive, or dangerous faults, i.e., minimizing system performance degradation, product deterioration, and equipment damage.
while improving human comfort and safety. FDD systems have also been studied under the terminology “condition monitoring,” whose purpose was to assist in the implementation of predictive maintenance as against the more common strategies of breakdown and planned maintenance (Davis 1998). Numerous studies relevant to HVAC&R equipment and systems are reviewed by Katipamula et al. (2001) and Comstock et al. (1999). A fundamental issue in model-based automated FDD is the identification of an accurate fault-free model of system performance as quickly as possible (i.e., from a monitoring period as short as possible) so as to subsequently use it for FDD purposes.

OBJECTIVE

Inverse modeling deals with identification of the system model structure based on measured data from an actual system in order to provide better control and prediction of the system response to external stimuli. It essentially consists of two distinct aspects: model formulation and parameter estimation. In the HVAC&R field, there has been a certain amount of effort put into developing proper physical models, especially during the last decade (for example, Gordon and Ng [2000] and Bourdouxhe et al. [1999]). Despite there being several publications (for example, IEA [1993]), the adoption of proper parameter estimation techniques has, to some extent, been neglected, partly because engineers are not well versed in statistical methods and partly because the added benefit of doing so was not clear. With the need for better M&V models as well as more robust automated FDD models, there is a growing body of such publications in the HVAC&R field, but much still remains to be done concerning how experimental measurements should be digested to ascertain statistically meaningful information.

An earlier paper by Reddy and Anderson (2002) described various statistical techniques pertinent to off-line parameter estimation of linear chiller models and illustrated these techniques using monitored data from a large centrifugal chiller. The primary objective of the present paper is to prepare the foundation for developing more scientific approaches in how to analyze and evaluate the statistical information contained in non-intrusive field monitoring (as against laboratory or intrusive testing) with the intent of either off-line or on-line model training in such application areas as M&V and FDD. We start by illustrating, by means of traditional statistical methods as well as resampling methods, the extent to which monitored field data are repetitive in their information content. Next, we briefly discuss notions relating to “information content” of data in various disciplines and how to evaluate whether a new datum is providing additional information to an already available data set. Subsequently, we propose and discuss two mathematical indices of information content relevant to our specific application. These are then applied to two field-operated chiller data sets, differing both in time scale of data collection as well as duration of monitoring in the framework of three different linear performance chiller models. We specifically discuss: (a) how these indices can provide insight into the initial length of the data set needed to initiate adaptive on-line model training, (b) how they can be used to determine when the monitored data do not provide any new information likely to modify the parameter estimates and the uncertainties of the linear models, and (c) how this initial data length required for proper parameter estimation depends on the model used.

STEADY-STATE CHILLER MODELS

A brief overview of inverse chiller performance models proposed in the HVAC&R area is provided below, separated into gray-box (or physical) and black-box (or empirical) categories.

Gray-Box Models

There are basically two physical chiller model formulations that have been proposed in the literature that are suitable for our purpose. The toolkit developed by Bourdouxhe et al. (1999) con-
sists of a set of computational tools based on physical algorithms for primary HVAC equipment. These are closer to lumped inverse component models, but the estimation of the parameters is nonlinear. The other formulation, and the one chosen for analysis in this study, is the Universal Thermodynamic Model proposed by Gordon and Ng (2000). The GN model is a simple, analytical, universal model for chiller performance based on first principles of thermodynamics and linearized heat losses. The model predicts the dependent chiller COP, defined as the ratio of chillers (or evaporator) thermal cooling capacity \( Q_{ch} \) by the electrical power \( P \) consumed by the chiller (or compressor) with specially chosen independent (and easily measurable) parameters such as the fluid (water or air) inlet temperature to the condenser \( T_{cdi} \), fluid temperature entering the evaporator (or the chilled water return temperature from the building) \( T_{chi} \), and the thermal cooling capacity of the evaporator. The GN model is a three-parameter model, which, for parameter identification, takes the following form:

\[
\left( \frac{1}{COP} + 1 \right) \frac{T_{chi}}{T_{cdi}} - 1 = a_1 \frac{T_{chi}}{Q_{ch}} + a_2 \frac{(T_{cdi} - T_{chi})}{T_{cdi}Q_{ch}} + a_3 \frac{(1/COP + 1)Q_{ch}}{T_{cdi}}
\]

(1)

where the temperatures are in absolute units, and the parameters of the model have physical meaning in terms of irreversibilities:

\[
a_1 = \Delta S = \text{the total internal entropy production rate in the chiller due to internal irreversibilities},
\]

\[
a_2 = Q_{\text{leak}} = \frac{1}{(mCE)_{\text{cond}}} + \frac{1 - E_{\text{evap}}}{(mCE)_{\text{evap}}} = \text{the rate of heat losses (or gains) from (or into) the chiller},
\]

\[
a_3 = R = \frac{1}{(mCE)_{\text{cond}}} = \text{the total heat exchanger thermal resistance, which represents the irreversibility due to finite-rate heat exchanger, and } E \text{ is the heat exchanger effectiveness.}
\]

The model applies to both unitary and large chillers operating under steady-state conditions. Evaluations by Reddy and Anderson (2002), Sreedharan and Haves (2001), and Jiang and Reddy (2003) have shown this model to be very accurate for a large number of chiller types and sizes.

If we introduce

\[
x_1 = \frac{T_{chi}}{Q_{ch}}, \quad x_2 = \frac{(T_{cdi} - T_{chi})}{T_{cdi}Q_{ch}}, \quad x_3 = \frac{(1/COP + 1)Q_{ch}}{T_{cdi}}, \quad \text{and} \quad y = \left( \frac{1}{COP} + 1 \right) \frac{T_{chi}}{T_{cdi}} - 1,
\]

Equation 1 assumes the following linear form:

\[
y = a_1 x_1 + a_2 x_2 + a_3 x_3.
\]

(2)

**Black-Box Models**

There are two broad categories of black-box models: classical and artificial neural networks (ANN) (see for example, Haykin [1999]). Though there are several papers on using ANN for HVAC&R equipment, we shall limit ourselves to the former in this paper. Whereas the structure of a gray-box model such as the GN model is determined from the underlying physics, the black-box model is characterized as having no (or sparse) information about the physical problem incorporated in the model structure. The model is regarded as a black box and describes an empirical relationship between input and output variables.

The commercially available DOE-2 simulation model (Winklemann et al. 1993) relies on the same variable as those for the physical model but uses a second-order linear polynomial model instead. This “standard” empirical model (also called a multivariate polynomial model), referred
to as the MP model by Reddy and Andersen (2002), has ten coefficients that need to be identified from monitored data.

\[ \text{COP} = b_0 + b_1 T_{cdi} + b_2 T_{ch} + b_3 Q_{ch} + b_4 T_{cdi}^2 + b_5 T_{ch}^2 + b_6 Q_{ch}^2 + b_7 T_{cdi} T_{ch} + b_8 T_{cdi} Q_{ch} + b_9 T_{ch} Q_{ch} \]  

(3)

These coefficients, unlike the three coefficients appearing in the GN model, have no physical meaning, and their magnitude cannot be interpreted in physical terms. Usually one needs to retain in the model only those parameters that are statistically significant, and this is done by step-wise regression.

The MP modeling approach has the advantage that it can be used in a routine manner. However, collinearity in regressors and ill-behaved residual behavior often justifies another empirical approach where the analyst transforms the variables so that a simpler (preferably linear) model is obtained that may overcome some of the problems of poor statistical estimation. This approach requires a certain amount of skill and is data specific. Such an approach, called the VT (variable transformation) model, has been proposed by Reddy and Andersen (2002) based on exploratory data analysis of a specific chiller used later in this study.

\[ \frac{Q_{ch}}{\log(P)} = c_0 + c_1 Q_{ch} + c_2 Q_{ch}^2 \]  

(4)

The model is linear in the parameters. Again, it should be emphasized that this model, Equation 4, is based on empirical data from one site only, and it might not be adequate for other chillers. For example, the influence of both the water temperatures has been found to be small in this instance and, hence, dropped from the model. This may not be at all valid both from the physics of chiller operation and how another chiller is operated.

**PARAMETER ESTIMATION**

Parameter estimation is the science of determining the most appropriate numerical values of the parameter estimates in Equations 2, 3, and 4, along with their associated uncertainty. Let \( X \) be the matrix containing the regressor data points as row vectors. Adopting matrix notation, the linear model can be formulated as follows:

\[ Y = X \cdot \beta + e \]  

(5)

where \( \beta \) is the vector of model parameters and \( e \) is the error term (i.e., the variation in \( Y \) unexplained by \( X \), which includes both model error and measurement errors in both \( X \) and \( Y \)). The ordinary least-squares (OLS) is the best method to use when there are no errors in \( X \), when there is no collinearity between the regressors, when model residuals have constant variance and are not patterned, and when the errors are normally distributed (Beck and Arnold 1977). In this case, the parameter vector is determined such that the sum of error squares function is minimized. This leads to the system of normal equations, provided matrix \((X^T X)\) is not singular (Draper and Smith 1981).

\[ b = (X^T X)^{-1} X^T Y \]  

(6)

where \( b \) is the vector of the least-squares parameter estimates of the model parameter set \( \beta \) and whose uncertainty \( \text{var}(b) \) is given by
where $\sigma^2$ = mean square error (MSE) of the model error terms.

FIELD-MONITORED CHILLER DATA SETS

Two data sets representative of large centrifugal chillers are used in this study.

Chiller #1 Data

The first data set consists of hourly field-monitored data over five months from a centrifugal chiller located in Toronto, Canada, from June through October. The analysis is performed using the following four variables (consistent with the GN, MP, and VT model formulations given by Equations 1, 3, and 4): (a) thermal cooling capacity $Q_{ch}$ in kWt; (b) compressor power $P$ in kWe; (c) supply chilled water temperature $T_{chi}$ in K, and (d) condenser water supply temperature $T_{cdi}$ in K. The data set used in the subsequent analysis contains 810 observations and is fully described in Reddy et al. (2001) and Reddy and Andersen (2002). From the time series plots of the four variables shown in Figure 1, we note that there is relatively little variation in the two temperature variables, while the load and power experience important variations. Since the chilled water flow rate is constant, we have chosen to perform the analysis with the following regressor set $[T_{cdi}, T_{chi}, T_{cho}]$ where $T_{cho}$ is the chilled water temperature leaving the chiller.

Chiller #2 Data

This is a 450 T centrifugal chiller located on the Drexel University campus. A comprehensive description of the steady-state data (consisting of 1126 sets of observations of 15-minute data over 14 days) is described in Reddy et al. (2001). Figure 2 depicts the time variation of the pertinent variables, namely, the condenser and evaporator fluid temperatures and the electrical power. The evaporator and condenser water flow rates can be assumed essentially constant since

\[
\text{var}(b)_{OLS} = \sigma^2(X^TX)^{-1},
\]

Figure 1. Time Series Data of the Four Measured Variables of Centrifugal Chiller #1 ($T_{chi} =$ Inlet Water Temperature to Evaporator (K); $T_{cdi} =$ Inlet Water Temperature to Condenser (K); $P =$ Electric Power Consumed by Chiller (kW); $Q_{ch} =$ Chiller Thermal Load (kW))
Figure 2. Time Series Plots of the Steady-State Filtered 15-Minute Data for Chiller #2
their variability (on the order of 1% to 2% of their mean value) is well within the measurement accuracy of the flow meters used. The chiller setpoint temperature denoted by $T_{ch0}$ exhibits only a 0.6°C range of variation between its maximum and minimum values, while the chiller inlet fluid temperature varies by about 4°C. Condenser outlet and inlet temperatures vary by about 11°C and 7.5°C, respectively. The values of load fraction range from 0.3 to 0.8, indicating that, on the whole, the data set does seem to contain acceptable variability in the chiller load. Of special note is the fact that the chiller was operated very uniformly for a long period between the observation range 420-600 (see Figure 2).

**ANALYSES TO EVALUATE REPETITIVENESS IN DATA**

We shall evaluate the extent to which field-monitored data are inherently repetitive in information content; in other words, to underline the fact that more field data does not necessarily mean more information. We shall investigate this issue in two ways:

1. **Traditional statistics:** One intuitive and simple way is to look at the histograms of the regressor variables since this would provide an indication of the variability in operating conditions to which the chiller is exposed. A uniform distribution would indicate good coverage of chiller operating conditions and vice versa. Figure 3 depicts such histograms for the three variables $T_{cdi}$, $T_{chi}$, and $Q_{ch}$ for Chiller #1. We note that $Q_{ch}$ values are fairly well distributed, while those for the two temperatures are not. For example, there are only a couple of data points for $T_{cdi} < 27.5^\circ$C. These points are likely to be influence points (Cook and Weisburg 1982), and whether these reflect actual operating conditions or are a result of either erroneous data or uncharacteristic chiller operation has to be determined by the analyst from physical (as against statistical) considerations. Additional insight into the extent to which the data collected are repetitive in nature can be gleaned by studying the joint occurrences. Table 1 shows these values for four bins of $T_{chi}$ and $Q_{ch}$ each and for two bins of $T_{cdi}$ (which exhibit

![Figure 3. Histogram of Number of Occurrences for the Three Regression Variables Using the Hourly Chiller #1 Data (Total of 810 Data Points)](image-url)
the least variability). It is obvious that there is great variability in how the individual bins are populated. For example, for the higher $T_{cdi}$ bin, there are large number of occurrences (95 and 106) for the middle two $Q_{ch}$ bins at the two extreme $T_{chi}$ bins. The extent to which a new datum point is bringing in additional information can be evaluated based on the number of data points already present in that particular bin. Ideally, population uniformity across bins, i.e., having a certain number of occurrences and no more within each bin (say, four to five in order to account for random noise in the data), would be a desirable situation offering sound parameter estimation with a minimum of data points. This is, in fact, what optimal design of experiments in a controlled setting strives to do.

2. Using resampling simulation methods to determine convergence and uncertainty bands of OLS parameter estimates. Specifically, we shall use the GN and VT models with the Chiller #1 data set and observe how the parameter estimates of both model formulations and their associated uncertainty bands converge as the length of data collection is increased.

Since the two models given by Equations 2 and 4 are linear in the parameters, OLS estimation is straightforward. It is important to emphasize that the purpose of applying OLS is to fit the models to the data and not for making statistical inferences about the model structure and parameters. Hence, some of the assumptions of OLS may be violated. Therefore, even though the estimating problem of the linear models may seem straightforward, one should be careful when one interprets the results. In general, it is recommended that some regression diagnostics be performed as described by Cook and Weisburg (1982). Such diagnostics would, however, be difficult to perform on a recursive or on-line basis. It is perhaps adequate to perform such diagnostics only once, on the preliminary model identified, which is done off-line. If the diagnostic results seem acceptable, no further model diagnostics are probably warranted.

Just as in the case of off-line parameter estimation, model robustness is greatly influenced by (a) colinearity between regressor variables and (b) serial correlation between all variables. We would expect differences between both model formulations (and in the indices representing the information content) since they have different correlation structure in their regressor sets (see Reddy and Andersen [2002]). Further, we would like to gain some broader insight into how these differences are affected by different time sequences of incoming data streams and not limit ourselves to the one sequence following which Chiller #1 data were collected.

<table>
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<tr>
<th>$Q_{ch}$ (kW)</th>
<th>$T_{cdi}$ (°C)</th>
<th>8.80-10.76</th>
<th>10.76-12.72</th>
<th>12.72-16.60</th>
<th>16.60-20.48</th>
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<tr>
<td>517.15-797.04</td>
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<tr>
<td>797.04-1076.93</td>
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Model robustness in terms of parameter variances can be investigated using resampling techniques. Resampling techniques (of which the jackknife and bootstrap methods are widely used) involve drawing samples either with or without replacement from the observed data set and performing analyses on them from which population-related estimators and prediction variance can be determined (Efron and Tibshirani 1982). The jackknife method, introduced by Quenouille in 1949 and later extended by Tukey in 1958, is a technique of universal applicability that allows confidence intervals to be determined of an estimate calculated from a sample while reducing bias of the estimator. The bootstrap method is similar but differs in that no groups are formed but the random samples are simply created by sampling with replacement from the observational data set (Davison and Hinkley 1997). Individual estimators deduced from such samples permit estimates and confidence intervals to be determined.

There are several numerical schemes for implementing the jackknife scheme. We have adopted the following:

a. a sliding window length of \( m \) data points is selected starting at point \( m_0 \);

b. the GN and VT parameter estimates are deduced for this data set;

c. steps (a) and (b) are repeated a large number of times—specifically 400 times in this analysis with the starting point \( m_0 \) being moved from 1 to 400;

d. mean values of the parameter estimates and their 2.5% and 97.5% percentiles are calculated from the 400 sets pertaining to window length \( m \);

e. steps (a) through (d) are repeated by incrementally changing the window length \( m \) from \( m = 20 \) to \( m = 400 \).

Chiller #1 data were used with the above scheme. Figures 4 and 5 depict the results of this analysis. Convergence along with acceptable 2.5 and 97.5 percentiles seems to require a minimum of 60 to 70 data points for the VT model parameters and close to 300 for the GN model parameters. The parameters of the VT model converge four to five times faster than the GN model, i.e., the physical model requires at least four to five times more data in order to obtain reasonably accurate parameter estimates. The parameter uncertainty of the two models, shown in Figures 4 and 5, is partly due to the ill conditioning of the GN model structure (see Reddy and Andersen [2002]), as well as due to the serial correlation in the data. The former effect is the reason why the convergence properties of the GN and VT models differ even when applied to the same basic chiller data set.

This important consequence of serial correlation is illustrated in Figure 6 for the GN model. The same procedure as previously is applied, but the \( m \) data points are no longer taken sequentially, but randomly, from the entire data set of 810 points with replacement (this is the bootstrap method). Consequently, most of the serial-correlation in the data is removed. Inspection of Figure 6 leads us to a completely different conclusion than previously. The number of observations (from 20 to 400 observations) seems to have no effect on the mean values of the model parameters nor on their variance (indicated by the 2.5 and 97.5 percentiles). In other words, using about 20 independent samples is just as good in terms of variance of parameter estimates as using 400 data points monitored continuously on-line! Comparing Figures 4 and 6 leads us to conclude that about 20 independent samples contain as much information as about 300 serial correlated observations in this case.

The above conclusion can be confirmed in a simple manner. From statistical sampling theory, the number of independent observations \( n' \) of \( n \) observations with constant variance but having a 1-lag autocorrelation \( \rho \) is equal to (Reddy and Claridge 2000)

\[
n' = n \cdot \frac{1 - \rho}{1 + \rho}.
\]
Figure 4. Convergence Properties for the Parameter Estimates of the GN Model Applied to Chiller #1 Data; for Each Plot, the Middle Line is the Calculated Mean and the Upper and Lower Lines Correspond to the 97.5 and 2.5 Percentiles; the X-Axis Shows the Window Size, and the Y-Axis Shows the Parameter Estimates $b_1$, $b_2$, and $b_3$.

Figure 5. Same as Figure 4 but for the Black-Box VT Model.
The 1-lag serial correlation among the basic variables for the Toronto chiller data is about 0.90 (see Reddy and Anderson [2002]), which results in \( \frac{n'}{n} = \frac{1}{19} \). Thus, the number of independent observations \( n' \) from \( n = 300 \) observations would be equal to about \( \frac{300}{19} = 16 \), which is consistent with the above observation that about 20 independent samples are sufficient. Though these numerical values are specific to the data set analyzed, the general behavior would apply to other non-intrusive field-monitored chiller data sets as well. This is an important insight with implications for both off-line and on-line performance modeling of field chillers in general.

**MEASURES OF INFORMATION AND INITIAL TRAINING DATA LENGTH**

**Theoretical Background**

Information science distinguishes between “message” and “information” (Cyganski and Orr 2001). While the former is the content of the transmitted data, the latter represents the conveyed knowledge that is in some way useful and not known previously. The concept of quantifying the information content of data is not new nor is it limited to one discipline. Broadly speaking, it is an “attempt to measure” (i.e., quantify or enumerate) the valuable information in a set or stream of data (Neumann and Krawczyk 2001). It finds application in areas such as information theory, statistics, and applied science, depending on the particular problem or intent of the stipulated problem. The basic underlying premise is that information is gained when uncertainty is reduced. For example, in information theory, the expected value of the information in a sample taken from a distribution (called the entropy of the distribution) is the average number of bits required to describe the sample (Gershenfeld 1999). The entropy is a maximum if the distribution is flat (since one knows nothing about the next point) and a minimum if the distribution is sharply peaked. In the design of experiments, the criterion based on which one determines...
whether a datum contains useful information or not is judged by whether it reduces the uncertainty in the parameter estimate vector $b$ given by Equation 6. Thus, the information content can be quantified as a function of the matrix $(X^TX)^{-1}$ following Equation 7. Rather than looking at a matrix, it is much more convenient to track a single scalar. Hence, though some information may be lost, some sort of “projection” or reduction in dimensionality is needed. Two such projections (which are to some extent analogous) are:

1. **Trace** of a matrix (see any pertinent statistical textbook):
   \[
   I_1 = \text{trace}((X^TX)^{-1})
   \]
   where the trace of a matrix is equal to the sum of the diagonal elements; $I_1$ of Equation 8 can be interpreted as the sum of the parameter variances up to a scaling factor.

2. Another measure of information is the **log of the mean of the determinant** (Beck and Arnold 1977).
   \[
   I_2 = -\ln(\text{det}(X^TX)/m)
   \]
   where $m$ is the number of observations used to calculate $(X^TX)$. Beck and Arnold (1977) showed that minimizing $\text{det}(X^TX)$ or, alternatively, minimizing $\ln(\text{det}(X^TX))$ also minimizes the uncertainty of the parameter estimator vector $b$.

Measure $I_2$ may be better than measure $I_1$ for two reasons. First, it has a clear physical interpretation in that it represents the hypervolume of the hyperellipsoid of the confidence region of the parameter estimates of the model (Beck and Arnold 1977). Second, the logarithmic transformation makes the numerical values more robust.

The above notions are relevant in the optimal design of experiments as well as during field monitoring. The analyst can tailor the experimental sequence so that a minimum number of experiments can be performed that will provide the necessary accuracy in the model parameter estimates. How these indices can be used for optimal experimental design for assurance testing of chillers during the factory witnessed test is described in a paper by Corcoran and Reddy (2003). Not so obvious, however, is the value of such measures of information in the context of non-intrusive field-monitored data where little can be done to select data in an optimal way. One advantage is that instead of tracking the uncertainty of the full set of model parameter estimates individually as more data are forthcoming (which is tedious), it is far simpler, both computationally and for decision making, to track only one overall measure. This aspect is further discussed later in this paper.

**Application to Chiller Data**

The set of 810 observations of Chiller #1 data has been used to calculate the measures of information $I_1$ and $I_2$ using Equations 8 and 9, respectively. The computation has been performed for three cases:

1. **Incremental window**, i.e., an ever-increasing window length to mimic the manner in which on-line data will be collected in actuality. Specifically, we started the computation with an initial set of ten data points, which was increased one observation at a time till the end of the data stream was reached (represented by the point $m_0 = 810$).

2. **Sliding window with 100 data points**, where the sliding window concept is similar to the generalized moving average concept used in time series analysis.

3. **Sliding window of 200 data points**.
How these two measures of information change with $m_o$ under the above three cases is shown in Figure 7. It is clear that both measures continuously decrease as more data are collected even though the data may be repetitive. However, the asymptotic nature of the curves indicates that there is a decreasing value to incoming data as $m$ increases. We see that the first 20 or 30 data points seem to bring in large amounts of “new” information, resulting in a steep initial drop in the plots, which gradually decreases in time. Also to be noted is the fact that though both the plots exhibit close similarities, the exponential scale of $I_1$ conveys a lack of robustness. Further, as expected, a sliding window of 100 data points shows more variability than one with 200 data points and with the incremental window case, indicating the importance of gathering data over a relatively large portion of the year. Finally, we note from the incremental window plots that a window size of about 100 data points would be an acceptable waiting period before initiating on-line training. It is obvious that in practice, one would implement a scheme where the information measure is tracked right from the onset of data collection, and a decision is made by the engineer of when to start on-line model training by inspecting the shape of the $I_2$ plot as it unfolds in time.

Figures 8a and 8b illustrate how the measure of information $I_2$ changes under incremental and sliding window scenarios for the basic variable set and the regressor set of the three linear chiller models. The earlier observation of using 100 data points to initiate model training is also supported in the framework of all three chiller models investigated. We note that the numerical value of $I_2$ is lowest for the VT model (indicating that it has the lowest parameter estimate uncertainty), while the GN model is poorest in its ability to provide sound parameter estimation. This is consistent with the strong correlation structure of the regressor matrix of the GN model as pointed out in an earlier paper by Reddy and Andersen (2002). On the other hand, from Figure 8b, pertinent to a sliding window of 50 data points, we note that the plot for the VT model is unstable, which indicates that the model is probably mis-specified (as was concluded from a residual analysis done by Reddy and Andersen [2002]).

Figure 7. Plots Illustrating How the Two Measures of Information $I_1$ (Shown on a Log Scale) and $I_2$ Change as More Data Are Incoming Under Both Incremental and Sliding Window Scenarios; the Basic Variable Set [$T_{c_d}$, $T_{ch}$, $T_{co}$] for Chiller #1 Has Been Used
Figure 8. Plots Illustrating How the Measure of Information $I_2$ Changes Under Incremental and Sliding Window Scenarios for the Basic Variable Set and the Regressor Set of the Three Linear Chiller Models When Applied to Chiller #1 Data
Recall that for Chiller #1 data, it was found that (1) the first 20 to 30 data points seem to bring in a large amount of “new” information, which gradually decreases in time, and (2) that a window size of 100 data points would be an acceptable waiting period before initiating on-line tracking. Exactly the same behavior can be noted from Figure 9 for Chiller #2 data, except that for this chiller it would be wiser to increase the window size to 200 data points, since there is more noise in this data set.

Another notable difference is that the time period 450-600 on the x-scale exhibits a sharp increase in the $I_1$ and $I_2$ plots (indicating a loss in information contained in this sliding window). This was verified to correspond to a region when the incoming data were essentially identical, denoting that the chiller was operating under steady-state constant conditions. Thus, not only was no new information brought in by the most recent data being collected, but the fixed window length selected eliminated earlier data that contained more variability in chiller operation. It is clear that another potential use of such measures is that they are capable of distinguishing different chiller operating regions.

APPLICATION AREAS

Length of Initial Data Set for Model Training

It is clear that on-line estimation pertains (1) to the situation when the chiller under study has just been installed or (2) when FDD-related instrumentation has just been implemented onto an existing chiller. One approach is to start with a chiller model initially trained off-line with manufacturer-provided data or from factory witnessed tests (Corcoran and Reddy 2003) for that particular chiller type and let these model parameter estimates adjust in time under...
actual chiller installation and performance. Another approach is to collect some data in the first few days (or weeks) of operation, make an initial estimation of the parameters off-line, and then let the values of these parameters be modified on-line as more performance data are forthcoming. In such a situation, the question is: How does one determine that the initial data collected are adequate to initiate on-line training? The solution is to track the information measures $I_1$ or $I_2$ on a continuous basis, as monitored data are forthcoming, and make a decision of when these measures of information show signs of asymptotic behavior. Either measure is equally appropriate, though measure $I_2$ is preferable, as discussed above. Since the chiller models are linear, the danger of local minimization (a problem encountered in nonlinear models and in ANN models) does not arise. Our bootstrapping analysis has revealed that though 20 independent data samples provide satisfactory predictions of model parameters and their variance, incoming data streams are likely to be highly correlated serially in time. Consequently, the initial data sequence needed to initiate model training will have to be larger than 20 samples. How much exactly depends not only on the specific circumstance but also to some extent on the correlation structure of the regressor matrix of the particular model. The VT model requires the least amount of data for sound parameter estimation (though the model itself is mis-specified) with the GN model requiring the most because of its strongly correlated regressor matrix. For the particular case of Chiller#1 data, which includes hourly data over several months, about 100 data points would be needed before starting training. On the other hand, for Chiller#2, the VT model requires about 100 while the GN requires about 200 data points. The indices $I_1$ or $I_2$ have been shown to be valid surrogates in indicating whether the data being collected are bringing in new information likely to decrease the uncertainty in model estimates and, thus, result in the identification of sounder and more representative model parameters.

**Scheme for Model Parameter Updating**

The issue of whether the on-line model parameters need to be constantly updated even when the most recent data do not bring in any “new” information can be approached from either a scientific or a practical point of view. The former view would be one that recommends updating the model parameters only when it is necessary to do so, which can be determined based on a measure such as the information measure $I_2$. However, from a practical point of view, this would mean computing $I_2$ at each incremental time step, making a determination of whether $I_2$ has changed sufficiently to warrant updating the model parameters, and doing so only if affirmative. Given (1) that the models are linear, (2) that computation time is negligible compared to the fastest sampling time adopted in HVAC&R thermal studies (on the order of one minute), and (3) that parameter estimates using the continuous updating scheme are no worse than those resulting from a selective updating scheme, it can be argued that it would be simpler to blindly update the model parameters irrespective of any changes in the index $I_2$. Which of these points of view will prevail in the longer run is an issue that M&V practitioners and control and service professionals involved in FDD will have to decide in the future based on practical considerations.

**M&V Applications**

Another potential benefit of looking at the information content of monitored data is that, in the future, M&V practitioners may conclude that the incremental cost of collecting data continuously over time, though small compared to the initial cost of installing and commissioning the

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*Note that an incoming datum may represent faulty chiller performance, and the FDD scheme must be able to discern such instances and not let the fault-free model parameters be trained with this datum. How to detect such faulty performance data is outside the scope of this paper. Here, we implicitly assume that all incoming data pertain to fault-free chiller operation.*
data collection system, is, however, not negligible. Such a consideration may lead them to
decide to collect all monitored data but only store data that brings in new information. Such a
scheme, if properly implemented, would reduce the effort related to the IT component (such as
maintaining a large database), as well as the analyst time and effort needed to handle and massage
large amounts of data without resulting in any loss of statistical information in the collected
data.

CONCLUDING REMARKS

The overall objective of this paper was to present the notion of “information content” of data
specifically as it applies to non-intrusive field-monitored performance data. This is a notion
widely used in various disciplines, and it may be advantageous if HVAC&R practitioners
involved in M&V and FDD were to adopt it to their work. Two indices are proposed whereby
one can evaluate whether a new datum is providing additional information to an already available
data set. In essence, new information is provided if the uncertainty in the model estimates is
reduced. Instead of tracking the uncertainty of the full set of model parameter estimates individu-
ally as more data are forthcoming (which is tedious), it is far simpler, both computationally and
for decision making, to track only one overall measure. Despite the fact that one cannot modify
the time sequence of non-intrusive field-monitored data collected, there are potential benefits of
analyzing monitored data in the framework of such measures of information as discussed and
illustrated in this paper.

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NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>COP</td>
<td>coefficient of performance</td>
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<tr>
<td>e</td>
<td>vector of model error</td>
</tr>
<tr>
<td>I₁, I₂</td>
<td>measures of information defined by Equations 8 and 9</td>
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<tr>
<td>m</td>
<td>number of data points used in the estimation</td>
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<tr>
<td>m₀</td>
<td>data point number in the data set</td>
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<td>n</td>
<td>total number of observation data</td>
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<tr>
<td>P</td>
<td>electric power consumed by the chiller</td>
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<td>Q_{ch}</td>
<td>thermal load on chiller</td>
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<td>T_{cdi}</td>
<td>fluid inlet temperature to the condenser</td>
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<tr>
<td>T_{chi}</td>
<td>fluid inlet temperature to the chiller or evaporator</td>
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<td>fluid outlet temperature leaving the chiller</td>
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REFERENCES


Comstock, M.C., B. Chen, and J.E. Braun. 1999. Literature review for application of fault detection and diagnostic methods to vapor compression cooling equipment, ASHRAE Research Project 1043-RP.


