

Uncertainty-Based Quantitative Model for Assessing Risks in Existing Buildings

T. Agami Reddy, Ph.D., P.E.

Member ASHRAE

Jason Fierko

Student Member ASHRAE

ABSTRACT

*Risk analysis involves three interrelated aspects, namely, risk assessment (characterization and estimation of potential adverse effects associated with exposure to hazards), risk management or mitigation (process of controlling risks or reducing their probability of occurrence by weighing alternatives and selecting appropriate action and also by putting in place response and recovery measures should an adverse phenomenon occur), and risk communication to the general public and concerned agencies. The objective of this paper is to propose a conceptual quantitative model for risk assessment in existing buildings that, while being consistent with current financial practice, would allow determination of expected annual monetary cost to recover from various risks. The methodology would thus provide guidance on identifying the specific risks that need to be managed most critically. The proposed methodology allows for the perceived importance with which **different stakeholders** in a building (for example, a building owner or the tenants) view the interaction between **vulnerable risk targets** (occupants, property damage, revenue loss) and **building elements** (such as civil, direct physical, cybernetic, mechanical and electrical system failure, and operation services) that are affected by different **hazard categories**. Each risk target is further subdivided into several sub-targets, while each hazard category is broken down into hazard events. The analysis involves (1) assigning conditional fuzzy values (with symmetric triangular membership functions) characterizing the perceived importance of different targets and subtargets to the concerned stakeholder, (2) multiplying them with the relevant binary applicability matrix (which is also stakeholder specific), thus, allowing subtargets to be*

mapped onto hazard categories, (3) multiplying them with historic hazard event probabilities (or absolute annual probability of occurrence of certain hazard events) that depend on such considerations as climate and geographic location of the city, location of building within the city, importance and type of building, and finally, (4) using industry-accepted building specific financial inputs (such as building replacement cost, net return on investment, number of occupants, insurance-related costs, etc.) to compute expected estimates of monetary risk (along with their uncertainty) to various hazards. We adopt a decision tree diagram approach for greater clarity in visualizing the process as well as the ease that it provides in performing the sequential calculations. An illustrative solved example pertinent to a large leased office building is presented and discussed to better illustrate the entire methodology. Logical improvements and extensions are also pointed out. The methodology proposed is of general relevance and is not meant exclusively for assessing risks due to extraordinary incidents.

RISK ANALYSIS: GENERAL BACKGROUND

Risk has different connotations in both everyday and scientific contexts, but all deal with the *potential* effects of a loss (financial, physical, etc.) caused by an undesired event or hazard. The analysis of risk can be viewed as a more formal and scientific approach to the well-known Murphy's Law (Wang and Roush 2000). Though different sources categorize them a little differently, the formal treatment of risk analysis includes three specific and interlinked aspects (NRC 1983; Haimes 1998; USCG 2001):

1. Risk assessment involves several activities such as identifying the sources and nature of the hazards (either natural or

T. Agami Reddy is a professor in the Civil, Architectural and Environmental Engineering Department of Drexel University, Philadelphia, Penn. **Jason Fierko** is with Ewing Cole, Philadelphia, Penn.

man-made), estimating the likelihood of their occurrence (i.e., quantifying them through subjective or objective probabilities), and, finally, evaluating the consequences (monetary, human life, etc.) were they to occur. Regardless of the type of potential loss, risk assessment can be one of two types: (i) *qualitative*, which is based on common sense or tacit knowledge of experienced professionals, and (ii) *quantitative*, which is based on adopting scientific and statistical approaches. Generally, the former is extensively used either during the early stages of a new threat (such as that associated with recent extraordinary incidents) or when the overall problem is so complex and uncertain in its cause and effects that quantitative methods yield close to meaningless results. Quantitative methods, on the other hand, provide great accuracy in applications where the hazards are reasonably well-defined in their character, probability of occurrence, and their consequences.

Quantitative risk assessment methods are tools based on accepted and standardized mathematical models that rely on real life data as their inputs. This information may come from a random sample, previously available data, or expert opinion. Risk assessment can be used to analyze the risk that is associated with a specific danger or to a whole gamut of hazards. The basis of quantitative risk assessment is that it can be characterized as the product of the probability of occurrence of an adverse event or hazard multiplied by its consequence. Since both these terms are inherently such that they cannot be quantified exactly, a major issue in quantitative risk assessment is how to simulate, and thereby determine, confidence bands of the uncertainty in the risk estimates. Very sophisticated probability-based statistical techniques have been proposed in the published literature involving traditional probability distributions in conjunction with Monte Carlo and bootstrap techniques (Haas et al. 1999) as well as artificial intelligence methods such as fuzzy logic (Hopgood 2001).

2. Risk management is the process of controlling risks, weighing alternatives, and selecting the most appropriate action based on engineering, economic, legal, or political issues. Risk management deals with how best to control or minimize the specific identified risks through remedial planning and implementation. These include (i) enhanced technical innovations intended to minimize the consequences of a mishap and (ii) increased training to concerned personnel in order to both reduce the likelihood and consequences of a mishap (USCG 2001). Thus, good risk management and control cannot prevent bad things from happening altogether, but they can minimize both the probability of occurrence as well as the consequences of a hazard. Risk management includes *risk resolution*, which narrows the set of remedial options (or alternatives) to the most promising few by determining their *risk leverage* factor. This measure of their relative cost-benefit is computed as the difference in

risk assessment estimates before and after the implementation of the specific risk action plan or measure divided by its implementation cost (Hall 1998).

Risk management also includes putting in place response and recovery measures. A major natural disaster occurs in the U.S. on an average of 10 times/yr with minor disasters being much more frequent (AIA 1999). Once such disasters occur, the community needs to respond immediately and provide relief to those affected. Hence, rapid-response relief efforts and longer-term rebuilding assistance processes have to be well thought out and in place beforehand. Such disaster response efforts are typically coordinated by federal agencies, such as the Federal Emergency Management Agency (FEMA), along with national and local volunteer organizations.

3. Risk communication can be done both on a long-term or short-term basis and involves informing the concerned people (managers, stakeholders, officials, public, etc.) as to the results of the two previous aspects. For example, at a government agency level, the announcement of a potential terrorist threat can lead to the implementation of certain immediate mitigation measures such as increased surveillance, while on an individual level it can result in people altering their daily habits by, say, becoming more vigilant and/or buying life safety equipment and storing food rations.

It is clear that all three aspects are interlinked since measures from one aspect can affect the other two. For example, increased vigilance can deter potential terrorists and thus lower the probability of occurrence of such an event. As pointed out by Haimes (1998), risk analysis is viewed by some as a separate, independent, and well-defined discipline as a whole. On the other hand, there are others who view this discipline as being a subset of **systems engineering** that involves (i) improving the decision-making process (involving planning, design, and operation), (ii) improving the understanding of how the system behaves and interacts with its environment, and (iii) incorporating risk analysis into the decision-making process. Because of the preliminary nature of this study, we shall simply adopt the narrower view of risk analysis, which can, nonetheless, provide useful and relevant tools to a variety of problems. Consequently, its widespread appeal has resulted in it becoming a basic operational tool across the physical, engineering, biological, social, environmental, business, and human sciences areas, which in turn has led to an exponential demand for risk analysts in recent years (Kammen and Hassenzahl 1999).

OBJECTIVE AND SCOPE

The objective of this study is to propose a conceptual quantitative model for risk assessment in existing buildings that is consistent with current financial practices, which would

provide guidance on identifying the specific risks that need to be managed most critically in the building under consideration. This would involve identifying and quantifying the various types and categories of hazards in typical buildings and proposing means to deal with their associated uncertainties. The methodology should also explicitly identify the vulnerabilities or targets of these hazards (such as occupant safety, civil and operating costs, physical damage to a building and its contents, and failure of one or several of the major building systems), as well as consider the subtler fact that different stakeholders of the building may differ in their perception as to the importance of these vulnerabilities to their businesses. Finally, the consequences of the occurrence of these risks have to be quantified in terms of financial costs consistent with the current business practice of insuring a building and its occupants. The quantitative risk assessment methodology should be simple, but not simplistic. Further, given the lack of previous studies of this sort, it should be flexible enough that it can be refined over time. The emphasis in this study is on developing the conceptual methodology rather than accurate quantification of the risks as they pertain to an actual building.

This study will be limited to assessing the risks in **existing commercial** buildings. For a new building in the design phase, there exist numerous additional construction and operation alternatives that are poorly delineated and understood and whose quantitative effect on the risk values are highly uncertain at best at this time. In addition, we shall choose a specific category among existing commercial building stock to focus on during this study. Huang and Franconi (1999) break up different commercial building types into twelve major categories: large offices, small offices, large retail stores, small retail stores, schools, hospitals, large hotels, small hotels, fast food restaurants, sit down restaurants, food stores (supermarkets), and warehouses. Of the above choices, this study focuses on large offices, though the risk analysis methodology suggested can be applied to any building category by suitable modification of the model inputs.

We start with a brief review of traditional areas where risk analysis has been applied, including a review of studies pertinent to buildings. Subsequently, we describe the methodology, the relevant inputs to the model, how uncertainty has been incorporated, and the calculation procedure. Finally, the methodology has been applied to a fictitious building to serve as a case study illustration.

RISK ANALYSIS: APPLICATION AREAS

Engineering

Recent world events have led leading American engineering societies (such as IEEE, ASCE, ASME, ASHRAE) as well as several federal and state agencies to form expert working groups with the mission to review all aspects of risk analysis as they apply to critical infrastructure systems. For example, in the area of buildings, certain specific aspects, such as IAQ,

building systems, and structural integrity, have become the focus of rather extensive risk management efforts by several universities, federal and national agencies, national laboratories, and private companies.

DeGaspari (2002) describes past and ongoing activities by ASME on managing industrial risk and quotes experts as stating that:

1. risk analysis with financial tools can benefit a company's bottom line and contribute to safety,
2. a full quantitative analysis can cost 10 times as much as a qualitative analysis, and
3. fully quantitative risk analysis provides the best bet for optimizing plant performance and corporate values for the inspection/maintenance investment while addressing safety concerns.

Lancaster (2000) investigates the major accidents in the history of engineering and gives reasons why they occurred. The book gives many statistics for different types of hazards and cost for each type of disaster. Additionally, chapters on human error are also included. Smith (2002) urges that design for terrorists requires a new way of thinking for engineers and the development of new protocols due to the unpredictable and illogical nature of the attack. This is complicated by the lack of scientific data to guide engineers as to how to counter it.

There is extensive literature on risk analysis as applied to nuclear power plants, nuclear waste management and transportation, as well as more mundane applications in mechanical engineering. A form of risk analysis that is commonly used in the engineering field is reliability analysis. This particular analytical approach is associated with the probability distribution of the time a component or machine will operate before failing (Vose 1996). Reliability has been extensively used in mechanical and power engineering in general and in the field of machine design in particular. It is especially useful in modeling the likelihood of a single component of the machine failing and then deducing the failure risk of several components placed in series or parallel. Reliability can be viewed as the risk analysis of a system due to mechanical or electrical failures, whereas traditional risk analysis in other areas deals with broader scenarios as described below.

Risk analysis has also been adopted in several other fields, for example, during building construction (McDowell and Lemer 1991). In this application, risk analysis deals with cost and schedule: (i) *cost risk analysis* is modeled as a discrete possible event, where the cost of the building is compared to a payback period, and (ii) *schedule risk analysis* deals with the connection between tasks that influence the construction time. Often, penalties must be paid if a building is not completed within the stipulated time period.

Business

Risk analysis has found extensive applications in the business realm, where several solutions may be posed, but only one is the best possible scenario. In a marketing application, a

sample can be taken from a random population, and through risk analysis and modeling, a marketing campaign can be designed. From a marketing standpoint, a company can identify the kind of campaign to which the public best responds and alter their marketing accordingly. Risk assessment techniques are also commonly employed in the business realm in order to help make important decisions, such as whether to invest in a venture or where to optimally site a factory or business. Such techniques are often rooted in financial modeling, where the risk is directly related to the monetary payoff in the end. There are four major categories of decisions in the business world that utilize risk assessment (Evans and Olson 2000): (a) acceptance or rejection of a proposal based on either net present value or internal rate of return; (b) selection of the best choice among mutually exclusive alternatives; for example, selecting a fuel source among wood, oil, or natural gas would be dependent on several factors such as price, availability, and growth rate; (c) selection of the best choice among non-mutual alternatives; (d) decisions containing a degree of uncertainty, which involve calculating the expected opportunity loss and return to risk ratios or creating decision trees and using Monte Carlo simulation techniques.

Public Health

Risk assessment is also commonly used when human health concerns are a factor. Haas et al. (1999) outline the primary areas where risk assessment is applied in health situations and give a process for performing a risk assessment. The two primary areas in health that utilize risk assessment are:

(a) *Human Health, Epidemiology, and Microbial Risk Assessment*

This area of study is concerned with the impact of exposure to defined hazards on human health. Epidemiology, which is a subset of human health, is the “study of the occurrence and distribution of disease and associated injury specified by person, place, and time” (Haas et al. 1999), while microbial assessment is concerned only with the disease and its opportunity to spread.

The risk assessment process in health situations consists of four steps: (i) *hazard identification*, which describes the health effects that are the result of human exposure to any type of hazard; (ii) *dose-response assessment*, which correlates the amount of time of the exposure to the rate of incidence of infection or sickness; (iii) *exposure assessment*, which determines the size and nature of the population that was exposed to the hazard and also how the exposure occurred, the amount, and the total elapsed time of exposure; and (iv) *risk characterization*, which integrates the information from the above steps to calculate the implications for the general public's health and calculates the variability and uncertainty in the assessment.

(b) *Ecological Risk Assessment*

Ecological risk assessment, on the other hand, refers to the destruction of the environment and the impact not only on

human life but on the fragile life in ecosystems. Standards for ecological protection may be more stringent than for human health protection in some cases.

Buildings

There are numerous publications dealing with risk analysis in buildings. A paper by McDowell and Lemer (1991) is the result of a committee meeting on building safety. It outlines the various risks associated with occupancy in a modern building. The motivation for the study was to provide higher safety in buildings through the use of risk analysis. However, the committee only proposes using risk analysis but does not actually apply the theory because of the lack of available information in the field. A paper by Harrington-Lynne and Pascoe (1995) reports on progress of research at the Building Research Establishment in England on the security of buildings from building design and urban planning aspects, their assessment and risk assessment strategies. Wright (1999) outlines the steps for creating a risk assessment profile for high occupancy buildings, such as multiple dwelling units, and factors related to the occurrence of a fire. A paper by Hale and Arno (2001) presents the results of an exhaustive survey to collect operational and maintenance data on over 200 power generation, power distribution, and HVAC items such as gas turbines and diesel engine generators, electrical switchgear, cables, circuit breakers, boilers, piping, valves, pumps, motors, and chillers. The results indicated that the maintenance quality level was a major predictor of equipment availability. Vine et al. (2000) identify a gamut of risk management opportunities and insurance loss reduction with energy-efficient and renewable energy technologies and procedures.

Recent events have spawned a totally new and extensive list of publications. The American Institute of Architects (AIA) has an extensive, frequently updated database of books, articles, audiotapes, and videotapes on a range of building security topics on their web page. This collection covers both new design and existing buildings and touches on risk assessment, management, and communication as well as disaster recovery and response. *HPAC Engineering* magazine (Ivanovich 2001) as well as members of the Panel on Energy Facilities, Cities, and Fixed Infrastructures operating under the U.S. National Academy of Sciences' Committee on Science and Technology for Countering Terrorism (Spielvogel 2002) have assembled a valuable list of relevant academy reports, technical papers, product lists, etc. Federal agencies (for example, USACE 2001) have prepared documents on protecting buildings and their occupants from airborne hazards. ASHRAE has assembled a study group on Health and Safety under Extraordinary Incidents that prepared a document (ASHRAE 2002) providing “initial guidance on actions that should be taken to reduce the health and safety risks of occupants.” Seminars on disaster preparedness involving security, emergency response, and loss minimization are also being offered by such organizations as the Association of Energy Engineers on a regular basis.

Software has been developed by Sandia National Laboratory (Hunter 2001) specifically to analyze risks for General Services Administration (GSA) buildings arising from natural disasters. This is the first risk-based approach to building management we have been able to locate. It allows assessing risks due to terrorism, natural disaster, and crime in federal buildings nationwide. It can also be adapted to other types of critical facilities such as embassies, school systems, and large municipalities. Another software tool is currently being developed by NIST (Chapman 2003), which will provide building/facility owners and managers alternative investment advice on risk mitigation strategies as a result of terrorist attacks. Other than these efforts, we have not been able to identify any quantitative risk assessment models that deal with existing buildings in an overall manner. The scope of the methodology proposed in this paper is broader (though it contains less specific and pertinent input data to operate the model) since a means of incorporating financial implications of specific hazards events is proposed.

METHODOLOGY SUGGESTED

There has been a certain amount of skepticism by policy and decision makers toward risk assessment models even when applied to relatively well-understood systems. The causes for this lack of model credibility have been listed by Haimes (1998) and include such causes as naïve or unrealistic models, uncalibrated models, poorly skilled users, lack of multiobjective criteria, overemphasis on computer models as against tacit knowledge provided by skilled and experienced practitioners, etc. These causes are very likely to apply to quantitative risk assessment models being developed for buildings; however, it is our opinion that they should not be taken as a deterrent in developing such models.

Stakeholders at Risk in Buildings

As stated earlier, the primary focus of this paper is medium to large office buildings though a similar approach can be adopted to other building types as well. A financial analyst setting up a portfolio for an individual takes several variables into account before deciding on the breakdown of investments. Of note to the analyst are considerations based on the individual's age and risk tolerance. Typically, a more aggressive investment strategy is suggested for a young person, and vice versa. This strategy parallels our methodology for acceptable risk in buildings. Similar to an individual saving for retirement, the person analyzing risk in a building must first consider the type of stakeholder (say, the owner or the tenant in a leased building scenario). The owner may be more concerned with the risk to the civil construction and to the basic amenities, which are his responsibilities, whereas the tenant may be more concerned with the cost of replacing the business equipment along with lost revenue should a deleterious event occur. Finally, both may be liable to be sued by the occupants if they were to be harmed. Hence, we need to start with the stakeholder.

There are several stakeholders in a building. They include anyone who has an interest in the design, construction, financing, insurance, occupancy, or operation of a building. This list includes, but is not limited, to the following groups of people: (i) building owner/landlord, (ii) building occupants/tenants, (iii) architects/engineers, (iv) local code officials, (v) township/city representatives, (vi) neighbors, (vii) contractors/union officials, (viii) insurance companies, and (ix) banks/financing institutions. For the purposes of this study, we shall consider only the first group—building owners. This study focuses on the risks associated with an occupied leased commercial building, not with the design, code compliance, and construction of such a structure.

The building owner or landlord is defined as the person who finances and operates the building on a daily basis. This person is concerned more with the physical building and the financial impacts on the operation of such a structure. The concerns of the building owner lie in the areas of operating costs and the physical building. For the purposes of this study, the building owner will not be a regular occupant of the building; rather this role will be performed from a remote location. On the other hand, the occupants of the building are defined as the people who work in the building on a daily basis. A typical occupant is a company that leases space from the building owner. Although the occupants are concerned with the physical building to some degree, this group is much more concerned with the well-being of its employees and the company-owned contents inside of the building. This group is also less sensitive to financial impacts on the operation of the building, since it is assumed that their lease rate is not sensitively tied to fluctuations in building operations cost. Additionally, the tenant can be viewed as a part-time occupant who has the option of leaving the building after the lease has expired. The situation where the occupant is also the owner of the building will not be considered in this risk assessment study, although the proposed model can be altered to fit the condition.

Hazard Categories and Affected Building Elements

Next, we need to distinguish between different building elements that can be damaged should different hazards occur. We have selected five building elements that are susceptible to the various hazards (or hazard events) that can befall a building. Further, these hazard events, though numerous, can be grouped into the following hazard categories (see Table 1):

- (a) Hazards to Civil (Building Specific)
 - (i) *Natural*: events such as hurricanes, earthquakes, tornadoes, floods, winter storms, electrical storms, high winds, earthquakes, tidal waves, and landslides. Hunter (2001) examined five of these natural hazards, but other hazards may be important in specific geographic locations.
 - (ii) *Intentional*: acts that are purposely committed with malicious intent and that are directly related to the

Table 1. Description of Different Hazard Categories that Impact Specific Building Elements

Building Element	Hazard Category	Description
Civil	Natural	Natural events that affect the civil construction of a building, such as earthquakes, floods, and storms
	Intentional	Actions that are purposely committed and designed to harm the physical building, such as bombings and arson
	Accidental	Actions that are not committed intentionally but have serious results, such as unintentional fires and accidents
Direct Physical	Crime	Actions that only affect the occupants and not the physical building, such as robbery or assault
	Terrorist	An act of terrorism that is intended to affect the occupants only, such as hostage situations
	Bio & IAQ	Contamination of air in order to harm building occupants
Cybernetic	Intentional	Sabotage, hacking in IT networks
	Accidental	Not committed on purpose but result in harm, such as computer crashes
MEP System	Accidental	The failure of mechanical, electrical, or plumbing systems, as well as telecom, fire safety equipment
Operation Services	Unanticipated	Impact of the fluctuations of utility prices and operation and maintenance that are required to operate the building

physical building, such as arson, bombings, and other extraordinary events.

- (iii) *Accidental*: acts that are not purposely committed but yield serious results. These acts include unintentional fires and other events.

(b) Hazards to Direct Physical (Individual)

- (i) *Crime*: acts that are directly related to the building occupant and perpetrated by someone with malicious intent. Examples include robbery and assault situations.
- (ii) *Terrorist*: acts perpetrated on a larger scale and include such events as hostage situations. Once again, these acts are directed solely at the building occupants, whereas bombings are categorized under civil hazards.
- (iii) *Bio/IAQ*: biological threats and indoor air quality concerns. These hazards are perpetrated through the building systems with serious effects on the occupants. Examples of these hazards include bio-terrorism and other indoor air quality concerns, whether natural or intentional.

(c) Hazards to Cybernetic Hazards

- (i) *Intentional*: computer hacking and industrial sabotage through the use of the computer system.
- (ii) *Accidental*: unintentional compromising of the network security system or unplanned crashes of the computer system.

(d) Hazards to MEP System

- (i) *Accidental*: mechanical and electrical systems, including security systems, fire alarms, building access, and

telecom systems. The reliability model can be adopted for this hazard.

(e) Hazards to Building Operation Services

- (i) *Unanticipated*: variations in fuel and energy prices and the impact that they will have on the general operation costs of the building. This analysis is more closely related to a sensitivity analysis and is impacted by the flexibility of the building systems.

Vulnerable Targets in a Building

Targets are building physical, human, and financial entities that are vulnerable to different building hazards (see Table 2). We propose the following three major categories of targets, which are further subdivided into subtargets:

- (a) *Occupants*, who are subject to risks, which are (i) short term: death, disability, (ii) long term: disability, burns, etc.
- (b) *Property Replacement*, which includes (i) physical building: building envelope; (ii) contents: furniture, equipment, computers, etc.; (iii) indoor environment: indoor air contamination and remediation; and (iv) building systems (mechanical and electrical): HVAC, elevators, lighting, etc.
- (c) *Revenue Loss*, which includes (i) operating cost, (ii) lost business, and (iii) cost of utilities.

Applicability Matrix

The hazards listed above have an impact on the stakeholders listed previously in different ways. One set of stakeholders may be more sensitive to a certain risk event than to others. The risk analysis methodology should explicitly consider this linkage between targets and subtargets to affected building

Table 2. Applicability Matrix from the Perspective of the Building Owner

Vulnerable Targets	Subtargets	Building Elements	Civil Structure (Building Specific)			Direct Physical (Individual)			Cybernetic		MEP System	Operation Services
			Natural	Intentional	Accidental	Crime	Terrorist	Bio & IAQ	Intentional	Accidental		
Occupants	Short-term	Hazard Categories	1	1	0	1	1	1	0	0	0	0
	Long-term		1	1	0	1	1	1	0	0	0	0
Property Replacement	Physical Building	Hazard Categories	1	1	1	0	0	1	0	0	0	0
	Contents		1	1	1	0	0	0	1	0	1	0
	Indoor Environment		1	0	1	0	0	1	1	0	1	0
	Building Systems		1	1	1	0	0	1	1	1	1	1
Revenue Loss	Operating Cost	Hazard Categories	0	1	1	1	1	0	0	0	1	0
	Lost Business		0	0	0	0	0	0	0	0	1	1
	Cost of utilities		0	0	0	0	0	0	0	0	0	1

Table 3. Importance Matrix (IM1) between Stakeholder (Assuming Building Owner) and Target Types. The Numbers Are Fuzzy Values (Which Sum to Unity) with Associated Uncertainty in Parenthesis Assuming Symmetric Triangular Membership). The Numerical Values Correspond to the Illustrative Example.

	TARGETS =====>		
	Occupant	Property Replacement	Revenue Loss
Stakeholder	0.1 (0.01)	0.6 (0.01)	0.3 (0.02)

Table 4. Importance Matrix (IM2) between Targets and Subtarget Types for Stakeholder (Assuming Building Owner). The Numbers Are Fuzzy Values (Which Sum to Unity) with Associated Uncertainty in Parentheses (Assuming Symmetric Triangular Membership). The Numerical Values Correspond to the Illustrative Example.

Subtargets	TARGETS =====>		
	Occupant	Property Replacement	Revenue Loss
Occupant/Short-term	0.9 (0.01)		
Occupant/Long-term	0.1 (0.01)		
Prop. Rep/ Physical Building		0.5 (0.1)	
Prop. Rep/Contents		0.1 (0.02)	
Prop. Rep/Indoor Environment		0.1 (0.02)	
Prop. Rep./Bldg System		0.3 (0.005)	
Revenue Loss/ Operating cost			0.4 (0.1)
Revenue Loss/Cost of Utilities			0.4 (0.1)
Revenue Loss/Lost Business			0.2 (0.03)

elements and associated hazard categories. We propose to perform this mapping by defining an applicability matrix that depends on the type of stakeholder for whom the risk analysis is being performed. The applicability matrix is binary in nature (i.e., numerical values can be 0 or 1, with 0 implying “not applicable” and 1 implying “applicable”). Table 2 depicts such an applicability matrix (AM) from the perspective of the owner of a leased commercial building. For example, the building owner may not view revenue loss vulnerabilities due to lost business or cost of utilities to be his/her responsibility. Hence, the corresponding cells have been assigned a value of zero for the most part). There may be some disagreement among practitioners in the particular manner in which we have chosen to assign these binary numbers in the various cells in such a table and even on how we have chosen to classify the various targets and hazards. However, these can be changed to suit the individual preferences while the overall risk methodology proposed in this paper is unaffected (though the numerical results may differ).

Importance Matrices

The importance with which a particular stakeholder views a specific target type or subtarget is perhaps best modeled by fuzzy theory (see Appendix A). Fuzzy numbers provided by the stakeholder are used along with their uncertainty characterized by a symmetrical triangular membership function. The importance matrix IM1 for stakeholders versus target type on

one hand and that for target type and subtarget (called IM2) are shown in Tables 3 and 4, respectively. From a building owner’s perspective, property replacement is more crucial than, say, long-term occupant hazards, and the fuzzy values of 0.6 and 0.1 shown in Table 3 reflect this perception. The tenant is likely to view the relative importance of these two targets differently, which was the reason for our initial insistence that one should start first and foremost with the concerned stakeholder. Further, the building owner is likely to be more concerned with the short-term, as against the long-term, exposure (since occupants change and it is more difficult to prove the owner’s culpability), which is translated into fuzzy values of 0.9 and 0.1, respectively, in Table 4. The numerical values have no factual or historic basis, nor have they been deduced from surveys of a specific stakeholder class. They are merely illustrative numbers for the purposes of this study. Note that these fuzzy numbers are **conditional**, i.e., they add up to unity at each level. The membership function is characterized by only one number (representing a plus/minus range around the estimate) since a symmetric triangular function is assumed in order to keep the data gathering as simple as possible. The actual input fuzzy data need to be refined over time and be flexible enough to reflect, first, the actual perception of a class of stakeholder and, second, that of a specific individual stakeholder depending on preferences, circumstances, and special concerns.

Table 5. Event Probabilities P_j (Absolute Probability of Occurrence per Year) for Different Hazard Events and Associated Costs. The Numerical Values Correspond to the Illustrative Example Discussed in the Text.

Affected Building Element	Hazard Category	Hazard Event	Probability P_j	Associated Cost C_j (\$/yr)
Civil Structure	Natural	Hurricane	0.005	I_d (building)
		Earthquake	0.0005	
		Tornado	0.001	
		Flood	0.01	
		Winter storm	0.0005	
	Intentional	Arson	0.005	
		Bombing	0.002	
		Terrorism	0.003	
	Accidental	Fire	0.005	
Others		0.000		
Direct Physical	Crime	Robbery	0.01	I_d (occupant)
		Assault	0.01	
		Homicide	0.005	
		Rape	0.005	
	Terrorist	Hostage	0.008	
		Hijacking	0.005	
		Murder	0.003	
	Bio & IAQ	Intentional	0.02	
		Accidental	0.01	
Sick building		0.03		
Cybernetic	Intentional	Hacking/outside	0.01	C_{cyb}
		Hacking/ Inside	0.02	
		Industrial sabotage	0.02	
	Accidental	Crash	0.01	
		Power outage	0.08	
		Power surge	0.07	
MEP Systems	Accidental	HVAC/Plumbing	0.003	$C_{M\&E}$
		Electrical	0.002	
		Telecom	0.001	
		Security	0.002	
		Fire alarm	0.001	
		BMS	0.002	
Increase in Operation Services	Unanticipated	Fuel price	0.01	$C_{O\&M}$
		Elec. Price	0.008	
		Utility cost	0.005	
		Labor cost	0.007	

Table 6. Building-Specific Financial Data Assumed in the Illustrative Example from the Perspective of the Building Owner

Description	Symbol	Assumed Values	Calculated Values
Building initial (or replacement) cost	C_I	\$15,000,000	
Net return on investment	ROI	15% per year	\$2,250,000/yr
Number of occupants	N_{occup}	500	
Total amount of insurance coverage against occupant lawsuits	C_{Law}	\$10,000,000	
Occupant hazard insurance premium	I_{OH}	\$200/occupant/year	\$100,000/yr
Building hazard insurance premium	I_{BH}	$(2\% * C_I)$ per year	\$300,000/yr
Insurance deductible • for building • for occupants	I_d	$(5\% * C_I)$ $(5\% * C_{Law})$	\$750,000 (bldg) \$500,000 (occupants)
Annual building maintenance and utility cost	$C_{o\&M}$	$(5\% * C_I)$ per year	\$750,000/yr
Replacement cost of MEP equipment	$C_{M\&E}$	\$3,000,000	
Cost to recover from computer software failure	C_{cyb}	\$50,000	

Note that the current methodology overlooks variability or uncertainty in these data

Hazard Event Probabilities

Hazard categories proposed have been described above and also summarized in Table 1. We need finer granularity in the hazard categories by defining hazard events since each of them need different risk management and alleviation measures. The various event categories adopted in this study are shown in Table 5. We note that natural civil hazards can result from five different events (hurricane, earthquake, tornado, flood, and windstorm). Though we do not claim that the list of events considered is comprehensive, this categorization should be adequate to illustrate the risk assessment methodology proposed. It is relatively easy to add, or remove, specific hazard events, or even regroup some of them, which adds to the flexibility of the proposed methodology.

Each of the events will have an uncertainty associated with it. These are best represented by hazard event probabilities or **absolute annual probability** (contrary to conditional probabilities, these will not sum to unity) of occurrence of certain hazard events and a distribution to characterize its uncertainty (see Appendix A). The absolute probabilities assigned to specific hazard events will depend on such considerations as climate and geographic location of the city, location of building within the city, importance, and type of building. These could be obtained through the research of historical records, as was done for the RAMPART database (Hunter 2001). In order to keep the assessment methodology simple, we have intentionally overlooked the uncertainty of these event probabilities, which would require more sophisticated analysis methods such as Monte Carlo methods (see for example, Haas et al. 1999). Numerical values of the hazard event probabilities used in this study are shown in Table 5. These are fictitious and are meant for illustrative purposes only.

Hazard Event Costs

The consequences, or cost implications, of the occurrence of different hazard events from the perspective of the stakeholder (in our case, the building owner) need to be determined in order to complete the risk assessment study. How we have chosen to consider these costs is summarized in the last column of Table 5 and is described below. Replacement costs for specific hazard events are difficult to determine, and more importantly, these costs are not reflective of the actual cost incurred by the building owner (unless he is self-insured). Most frequently, the building owner insures the building along with its contents and occupants with an insurance company to which he pays annual premiums (say, I_{BH} for civil construction and I_{OH} for occupants) whether or not a hazard occurs (see Table 6). However, the actual additional cost faced by the building owner when civil and/or direct physical hazards do occur is actually the insurance deductible I_d . On the other hand, financial risks due to accidental MEP and cybernetic hazards ($C_{M\&E}$ and C_{cyb}) are considered to be direct expenses the owner incurs whenever these occur. The monetary consequence of risk due to an unanticipated increase in operation services (maintenance, utility costs, etc.) is taken to be $C_{o\&M}$, which can be assumed to be a certain percentage of the total building cost that is spent yearly on operations, maintenance, and utility costs. Most of the above costs need to be acquired from the concerned stakeholder.

Computational Methodology

A decision tree is a graphical representation of all the choices and possible outcomes available in a risk assessment study. It is widely used in quantitative risk analysis (see, for example, Smith 1997; Haines 1998; Wang and Roush 2000). The outcomes of each decision in the tree are assigned probabilities according to existing factual information or profes-

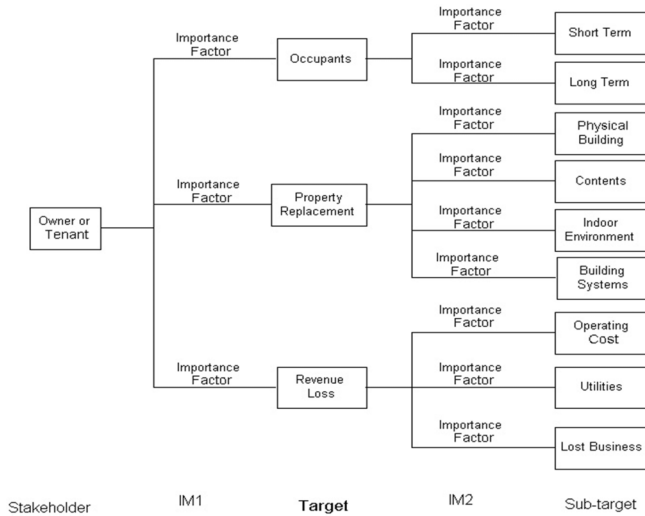


Figure 1 Overall risk assessment tree diagram by target category.

sional input. Levels of the tree can be created according to how involved the decision process is and how many outcomes are possible. The decision tree creates a highly visual way of considering alternatives and allows easy manipulation of the data if input values need to be changed. We start with the stakeholder and his/her perceived importance toward various targets (characterized by the IM1 matrix) and subtargets (characterized by IM2 matrix). This is shown in Figure 1. Next these subtargets are mapped onto the hazard categories using the binary information contained in the Applicability Matrix AM (Table 2). Finally each of the affected building elements and the associated hazard categories are made to branch out into their corresponding hazard events with the associated absolute probability values (as shown in Figure 2). Once a probability is assigned to a specified risk event, a monetary value also needs to be associated with it that is representative of the cost in undoing or repairing the consequences of that event. By multiplying the monetary value and the probability (last two columns of Table 5), a characterization of the expected value of risk becomes apparent. Addition of all the outcome values will give an overall characterization of the building. Areas of high probability or high cost can thus be easily identified and targeted for improvement.

Mathematically, the above process can be summarized as follows:

Expected annual monetary cost for a given target k to recover from hazard event j of a specific hazard category i

$$= (IM1_k \cdot IM2_{k,l}) \cdot AM_{l,i} \cdot (P_j) \cdot C_j \quad (1)$$

where i is the subscript denoting the hazard category, j the hazard event, k the target, and l the subtarget.

Multiplication rules for fuzzy numbers are relatively simple and are described in several texts (for example,

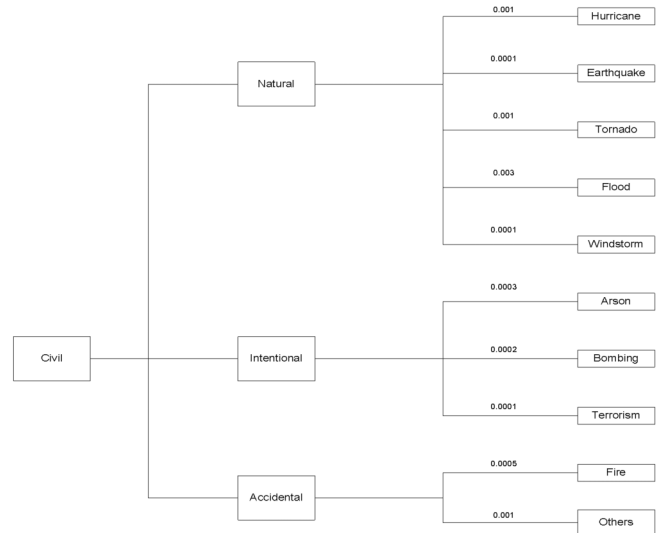


Figure 2 Tree diagram for various hazards affecting the civil element (numerical values of absolute probabilities correspond to the numerical illustrative example—see Table 5).

McNeill and Thro 1994). This is required for considering the propagation of uncertainty through the various sequential stages of the computation. Thus, in conjunction with computing the estimate of a risk, a range of numbers will also be computed reflective of the perceived importance to the stakeholder vis-à-vis specific targets and subtargets.

ILLUSTRATIVE EXAMPLE OF COMPUTER MODEL

A hypothetical solved example will be presented in this section to illustrate the entire methodology. We shall assume a commercial building with 500 occupants with the numerical values for AM , $IM1$, $IM2$, and P_j shown in Tables 2 through 5. Financial inputs and assumptions are shown in Table 6. The replacement cost of the building (C_l) is assumed to be \$15,000,000 with the ROI for the building owner to be 15% per year. His gross annual income is assumed to be \$3,400,000 per year, while his annual expenditures include \$300,000 (or 2% · C_l) as building hazard insurance premium, \$100,000 per year (or \$200 per occupant) as occupant insurance premium, and \$750,000 (or 5% · C_l) as building maintenance and utility costs. MEP replacement cost is estimated to be \$3,000,000, and cost to recover from a complete computer software failure is estimated to be \$50,000. The insurance deductibles for civil structure and occupants are 5% of C_l and C_{Law} (where C_{Law} is the total insurance coverage against occupant lawsuits), namely, \$750,000 and \$500,000, respectively.

ILLUSTRATIVE EXAMPLE OF COMPUTER MODEL

The results of the risk assessment are shown graphically in Figure 3 and in tabular form in Table 7. At the whole build-

Table 7. Monetary Risks on the Five Building Elements Affected by the Various Hazards (\$/yr)

Building Element	Mean	Lower Limit	Upper Limit
Civil	\$17,895	\$10,170	\$25,620
Direct Physical	\$24,260	\$13,380	\$35,140
Cybernetic	\$2,190	\$1,410	\$2,970
M/E Failure	\$15,840	\$6,270	\$25,410
Operations	\$8,100	\$2,925	\$13,275
Total	\$68,285	\$34,155	\$102,415

ing level, the building owner spends \$1,150,000 per year, with a net income of \$2,250,000. The monetary mean value of the total risks is \$68,285, i.e., about 3.0% of his net income. The owner may decide that this is within his/her tolerance threshold and do nothing. On the other hand, he/she may calculate the risk based on the upper limit value of \$102,415 shown in Table 7 and decide that 4.6% is excessive. In this case, he/she would want some direction as to how to manage the risk. The analysis at the affected building element level (see Figure 3) reveals that “Direct Physical” has the highest monetary risk, followed by civil and MEP failure. He/she consults Figure 4 and determines that Bio/IAQ has the highest monetary risk, which, from Figure 5, can be attributed to the possibility of occupant lawsuits complaining about sick building syndrome and to intentional hazard events. The building owner can then take necessary risk management actions: pay for additional security or implement certain technical measures (change amount of outdoor air intake, replace air filters with high quality filters, make control modifications to the VAV system, etc.) to alleviate this risk. Which specific measure would be most cost-effective to implement does not fall under the purview of risk assessment but under risk management and control, which is beyond the scope of the methodology proposed here.

CONCLUSIONS AND FUTURE WORK

We have proposed a methodology for performing a comprehensive quantitative risk assessment for existing buildings in general, though the focus of this paper was on leased commercial office buildings. The methodology involved creating a matrix of hazards and vulnerabilities from the point of view of a particular stakeholder and then generating the relevant tree diagram for performing the fuzzy-based risk assessment. The risk assessment yields expected annual monetary costs to recover from different hazard categories and events. Though disagreement is possible in the manner in which the disaggregation of hazards and vulnerabilities has been done, the methodology is general and flexible enough to accommodate changes. This study proposed a methodology but did not research into determining numerical values of the various specific inputs needed to use the model.

Breakdown by Affected Building Element (Mean Value- \$/yr)

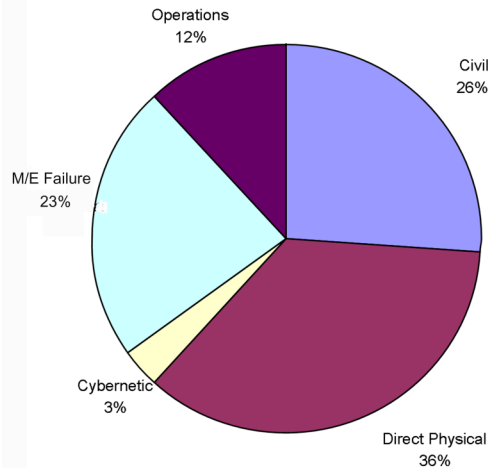


Figure 3 Risk assessment results for the illustrative example at the building element level.

Dollar Impact According to Hazard Category

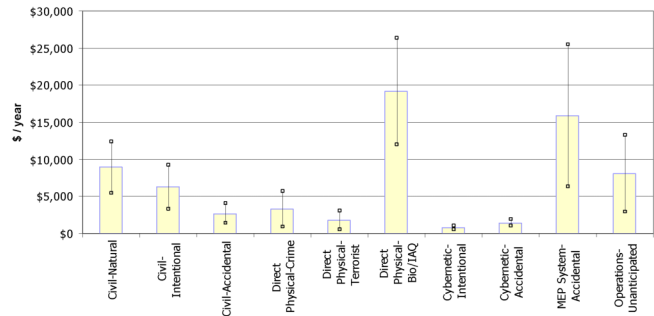


Figure 4 Risk assessment results for the illustrative example at the hazard category level. Uncertainty bands are also shown.

Several improvements and extensions need to be made to the proposed methodology before it can be applied in a meaningful, routine, and widespread manner:

- (a) First, its overall relevance needs to be assessed by practitioners and insurance agencies.
- (b) The methodology requires that the stakeholder provide input values that characterize the perceived importance of specific targets and subtargets. Surveys of different stakeholders should be conducted to determine the extent to which this perception differs from one stakeholder group to another as well as from one individual to another within the same stakeholder group.
- (c) The analysis also requires that event probabilities and remediation costs be known or estimated. Such database repositories of identified and evaluated risk information for different hazard types for different building

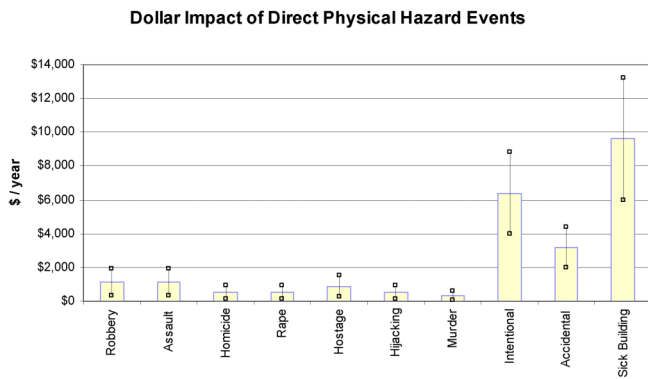


Figure 5 Risk assessment results for the illustrative example at the hazard event level for the direct physical hazard category. Uncertainty bands are also shown.

types, ages, and location need to be assembled along with realistic cost values for the analysis to be meaningful. This requires extensive literature research and scrutiny of available historic data. Further, responsible individuals need to update these databases over time. The existing RAMPART database (Hunter 2001) is a logical starting point.

- (d) The financial aspect of this analysis needs to be improved. Aspects such as mortgage payments, depreciation, etc., need to be explicitly considered in such a methodology.
- (e) This project focused solely on existing commercial buildings. Changes in the methodology will have to be made to tailor this risk assessment to other types of buildings, such as multi-person dwelling units, manufacturing plants, and public institutional buildings, where risk considerations may be different. Extension to new buildings is also an important and necessary activity.
- (f) The quantitative methodology may need to be refined in several ways. The additional benefits of assuming more sophisticated membership functions for the fuzzy conditional probabilities should be evaluated along with whether the corresponding survey data can support the additional complexity. Another issue is the uncertainty in the absolute event probabilities. These have been incorporated as point estimates in the present procedure. Mathematical rigor requires assigning probability distributions to each event probability (the Poisson distribution is often used to model such rare events) and determining the uncertainty ranges of the expected monetary risks to different hazards by Monte Carlo simulation methods. The benefits that this refinement would bring also need to be evaluated by future studies.
- (g) Finally, this paper only focuses on the risk assessment aspect of the entire risk analysis problem. In order for this tool to have overarching practical value to building owners, professionals, and concerned government agen-

cies, the scope of this study should be expanded to include the other aspects as stated in this paper, namely, risk management, communication, and response and recovery measures to be implemented.

ACKNOWLEDGMENTS

Insightful and detailed comments by Professor J. Mitchell and Dr. R. Hunter have been greatly beneficial in general formulation of the model approach and in improving this paper. Advice and suggestions on how to incorporate financial and natural risks were proffered by Drs. C. Haas, J. Mullin, and A. Zerva. Finally, the able assistance of C. Panjapornpon in programming the model into a user-friendly software program is appreciated.

NOMENCLATURE

- AM = applicability matrix (binary) between subtargets and hazard categories
- C_{cyb} = fees charged by computer software consulting company in the event of a complete computer system failure
- C_I = building initial (or replacement) cost
- C_j = cost for repairing the consequences of a specific hazard event
- C_{Law} = total amount of insurance coverage against occupant lawsuits
- $C_{M\&E}$ = accidental replacement cost of MEP equipment
- $C_{o\&M}$ = building operation, maintenance, and utility cost
- I_{BH} = building hazard insurance per year
- I_d = insurance deductible
- I_{OH} = occupant hazard insurance per year
- $IM1$ = importance factor matrix between stakeholder and target types
- $IM2$ = importance factor matrix between target types and subtargets
- N_{occup} = number of occupants in building
- P = absolute hazard event probability per year
- ROI = return on investment

Subscripts

- h = building element affected by the hazard
- i = hazard category
- j = hazard event
- k = target
- l = subtarget

REFERENCES

- AIA. 1999. *Guidelines for Disaster Response and Recovery Programs*. Washington, D.C.: American Institute of Architects.
- ASHRAE, 2002. *Risk Management Guidance for Health and Safety under Extraordinary Incidents*, report prepared

- by the ASHRAE Presidential Study Group on Health and Safety under Extraordinary Incidents. Atlanta: American Society of Heating, Refrigerating and Air-Conditioning Engineers, Inc.
- Chapman, R.E. 2003. *Cost-Effectiveness Tool for Evaluating the Management of Terrorist Risks*. Gaithersburg, Md.: Building and Fire Research Lab, National Institute of Standards and Technology.
- DeGaspari, J. 2002. Risky Business. *ASME Mechanical Engineering Journal* July: 42-44.
- Evans, J., and D. Olson. 2000. *Statistics, Data Analysis, and Decision Modeling*. Upper Saddle River, NJ: Prentice Hall.
- Haas, C., J. Rose, and C. Gerba. 1999. *Quantitative Microbial Risk Assessment*. New York: John Wiley and Sons.
- Haimes, Y.Y. 1998. *Risk Modeling, Assessment and Management*. New York: John Wiley and Sons.
- Hale, P.S., and R.G. Arno. 2001. Survey of Reliability and Availability Information for Power Distribution, Power Generation, and HVAC Components for Commercial, Industrial, and Utility Installations. *ASHRAE Transactions* 107(2).
- Hall, E.M. 1998. *Managing Risk*. Reading, MA: Addison-Wesley.
- Harrington-Lynne, J., and T. Pascoe. 1995. A Strategy for Security of Buildings. *IEEE Transactions*, 0-7803-2627-X/95.
- Hopgood, A.A. 2001. *Intelligent Systems for Engineers and Scientists*, 2d edition. Boca Raton, FL: CRC Press.
- Huang, J., and E. Franconi. 1999. *Commercial Heating and Cooling Loads Component Analysis*, Office of Building Technology, Berkeley, CA.
- Hunter, R. 2001. RAMPART Software. Sandia National Laboratories, <<http://www.sandia.gov/media/NewsRel/NR2001/rampart.htm>>.
- Ivanovich, M. 2001. *Quick Assessment of Resources and Advertisers for Disaster Planning and Response Publications*. HPAC Engineering magazine.
- Kammen, D.M. and Hazzenzahl. 1999. *Should We Risk It?* Princeton University Press.
- Lancaster, J. 2000. *Engineering Catastrophes: Causes and Effects of Major Accidents*. Cambridge: Abington Publishing.
- McDowell, B., and A. Lemer. 1991. *Uses of Risk Analysis to Achieve Balanced Safety In Building Design and Operations*. Washington, D.C.: National Academy Press.
- McNeill, F., and E. Thro. 1994. *Fuzzy Logic, A Practical Approach*. Boston, MA: AP Professional.
- NRC. 1983. *Risk Assessment in the Federal Government: Managing the Process*. National Academy of Sciences, National Research Council, National Academic Press, Washington, D.C.
- Smith, D.J. 1997. *Reliability, Maintainability and Risk*, 5th edition. Butterworth Heineman.
- Smith, E.M. 2002. Designing for Sabotage. *ASME Mechanical Engineering Journal* September: 40-44.
- Spielvogel, L. 2002. Personal communication, May.
- USACE. 2001. *Protecting Buildings and Their Occupants from Airborne Hazards*, report TI 853-01, prepared by the U.S. Army Corps of Engineers, Engineering and Construction Division, Directorate of Military Programs, Washington, D.C.
- USCG. 2001. *Risk-Based Decision-Making*, United States Coast Guard, <<http://www.uscg.mil/hq/g-m/risk/>>.
- Vine, E., E. Mills, and A. Chen. 2000. Energy Efficient and Renewable Energy Options for Risk Management and Insurance Loss Reduction. *Energy*, vol. 25, pp.131-147.
- Vose, D. 1996. *Quantitative Risk Analysis, A Guide to Monte Carlo Simulation Modeling*. New York: John Wiley and Sons.
- Wang, J.X., and M.L. Roush. 2000. *What Every Engineer Should Know about Risk Engineering and Management*. New York: Marcel Dekker.
- Wright, M. 1999. *High Occupancy Building Risk Assessment Toolkit*. London: Fire Research And Development Group.

APPENDIX A: SOURCES OF UNCERTAINTY AND MODELING APPROACHES

Because of the probabilistic nature of most problems, any risk analysis consists not only of determining point estimates of the various risks but also specifying their associated distributions or range of feasible values. The latter is often characterized by terms such as “uncertainty” and “variability.” Haimes (1998) discusses the prevailing confusion that exists in how these two terms are defined and used by different people and proposes a taxonomy of uncertainty that combines both these sources of ambiguity: incomplete knowledge and stochastic variability. He proposes a simple working definition: “uncertainty is the inability to determine the true state of affairs of a system.” A succinct classification and description is provided here as a brief background to the following sources of uncertainties:

- (a) Purely stochastic variability where the ambiguity in outcome is inherent in the nature of the process. Examples involve coin tossing or card games. These processes are inherently random (either on a temporal or spatial basis), and whose outcome, while uncertain, can be anticipated on a statistical basis.
- (b) Ignorance or lack of complete knowledge of the process that results in model deficiencies (use of surrogate variables, excluded influential variables, improper functional form, model approximations, etc.).
- (c) Inaccurate measurement due to instrument or sampling errors.
- (d) Cognitive vagueness involving human linguistic description. For example, people use words such as tall/short or very important/not important that cannot be quantified exactly.

The traditional approach is to use probability theory along with statistical techniques to address (a), (b), and (c) types of uncertainties. The stochastic variability (sources b and c) can be diminished by taking additional measurements, by using more accurate instrumentation, by better experimental design, and by acquiring better insight into specific behavior with which to develop more accurate models. Several authors apply the term “uncertainty” to only these two sources. Some authors (for example, Haas et al. 1999) make the distinction between these and (a) above by using the term “variability” to denote processes in category (a) where no amount of additional measurements can reduce the inherent uncertainty.

The source of uncertainty (d) is often modeled by fuzzy logic or possibility theory (see, for example, Hopgood 2001). Fuzzy logic is best defined as “a language that allows one to translate sophisticated statements from natural language into

mathematical formalism” (McNeill and Thro 1994). Since people answer questions in terms that are not always precise, the application of fuzzy logic allows a degree of linguistic uncertainty or vagueness to be associated with the meaning of a specific answer. This type of mathematical representation captures human thinking, decision making, and speech in a manner that has been shown to be more flexible and convenient to solve certain types of problems involving human interaction. A fuzzy number is characterized by an estimate along with a degree of fuzziness captured by a membership function. The simplest type of membership function is the symmetric triangular function characterized by one number representative of the range or confidence intervals of the estimate. Fuzzy numbers can be arithmetically manipulated creating a fuzzy product following well-established and relatively simple rules.